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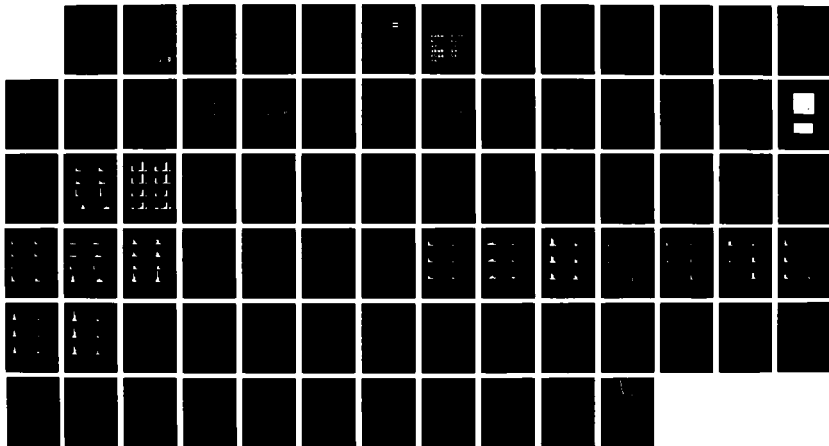
MASK MATCHING FOR LINEAR FEATURE DETECTION(U) MARYLAND
UNIV COLLEGE PARK CENTER FOR AUTOMATION RESEARCH
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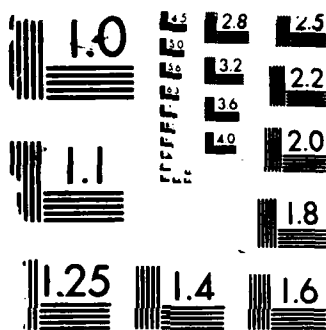
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MASK MATCHING FOR LINEAR FEATURE DETECTION

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ABSTRACT

This paper describes a set of masks that can be used in extracting thin linear features from images. A probabilistic analysis of mask matching for "white noise" images is given, and results of some experimental studies are also presented.

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1. Introduction

Mask matching is a well known procedure [1] in which the detection of specific features in an image is carried out by matching a set of templates or "masks" with the neighborhood of each pixel in the image.

This paper describes a set of 3×3 binary masks that can be used to extract linear features from an image. The masks are used to assign various labels to each pixel (each label corresponding to a particular mask), and to associate with each label a set of confidence measures based on the homogeneity of the foreground and background in the mask, and the difference between them. The idea of this approach is to record a large set of useful information at the pixel level, in order to efficiently make use of it at the later stages of the linear feature detection process.

In Section 2, we introduce the set of masks and the confidence measures. In Section 3, the data structures and the algorithm used for the implementation of the masks are discussed.

Section 4 contains empirical results obtained by applying the masks to several image samples. The principal conclusion drawn from these results is that it is difficult to decide which matched masks are part of a linear feature by simple thresholding of the confidence measures. However, it is shown in a companion report [2] that by using compatibility relations between masks at neighboring pixels, good linear feature extraction performance can be obtained.

Section 5 gives a probabilistic analysis of the frequency of matches and their expected "robustness" for specific masks and classes of masks in "white noise" images. These results may help indicate whether or not a given image region should be considered "interesting", as regards frequency of occurrence of "line-like" masks, for example.

2. The Masks and the Confidence Measures

2.1. Preliminary Remarks

By applying "mask matching" to an image, we mean finding those $N \times N$ image neighborhoods that match specific masks. The matching procedure is described later in more detail, but basically, matching occurs when there exists a threshold dividing the N^2 pixels into two groups, called dark and bright, such that the pattern of black and white pixels is exactly the same as a given mask.

Our goal is to extract "thin" linear features (1-2 pixels wide) from an image. For this task, masks having $N = 1$ or $N = 2$ are obviously too small. On the other hand, higher values of N result in much too large a set of masks. $N = 5$, for example, implies a set of $2^{25} = 32M$ possible binary masks, so even if only about 20% of these were selected as the subset of linear feature masks, the number of cases would still be far beyond practical levels. Even $N = 4$ implies $2^{16} = 64K$ masks, which is still very large. We have used $N = 3$ to reduce the set of masks to a reasonable size: $2^9 = 512$.

A representation for a 3×3 neighborhood is shown in Figure 1a. Note that except for the center pixel (referred to as CP hereafter), the number assigned to each pixel is the angle it forms with the CP (measured counterclockwise from the positive x -axis) divided by 45° .

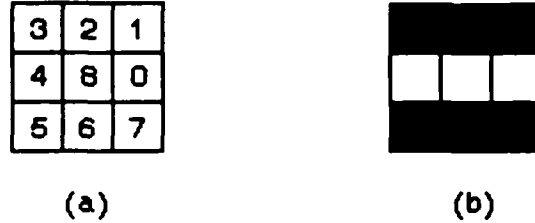


Figure 1. Representation for a 3×3 neighborhood. (a) Ordinal numbers of pixels. (b) The mask with $mask_number = 273$.

Each of the possible $2^9 = 512$ binary masks is assigned a mask number according to the formula

$$mask_number = \sum_{i=0}^8 b_i 2^i$$

where the value of b_i for the i^{th} pixel is either 0 (dark) or 1 (bright). (See Figure 1b, for example.) Note that we have $mask_number \geq 256$ iff the mask has a bright CP. Moreover,

$$mask_number(bright\ CP) + mask_number(dark\ CP) = 511$$

holds for all pairs of "complementary" masks (i.e., masks having the same pattern but with dark and bright exchanged).

A list of all possible 3×3 binary masks (in increasing order of their mask numbers) appears in Appendix A.

2.2. The Linear Feature Masks

We now introduce a set of masks corresponding to various possible configurations that could occur in a 3×3 neighborhood that intersects a linear feature. The masks are grouped into the following classes:

Edge, Line, Wide-Line, Right-Angle, Acute-Angle, End-Point, Corner, L-Line, T-Junction, X-Junction, and Y-Junction.

The dark CP masks belonging to each class are shown in Figure 2. The remaining dark CP masks in the set are obtained by rotating each of the masks shown in Figure 2 by 90° , 180° and 270° .

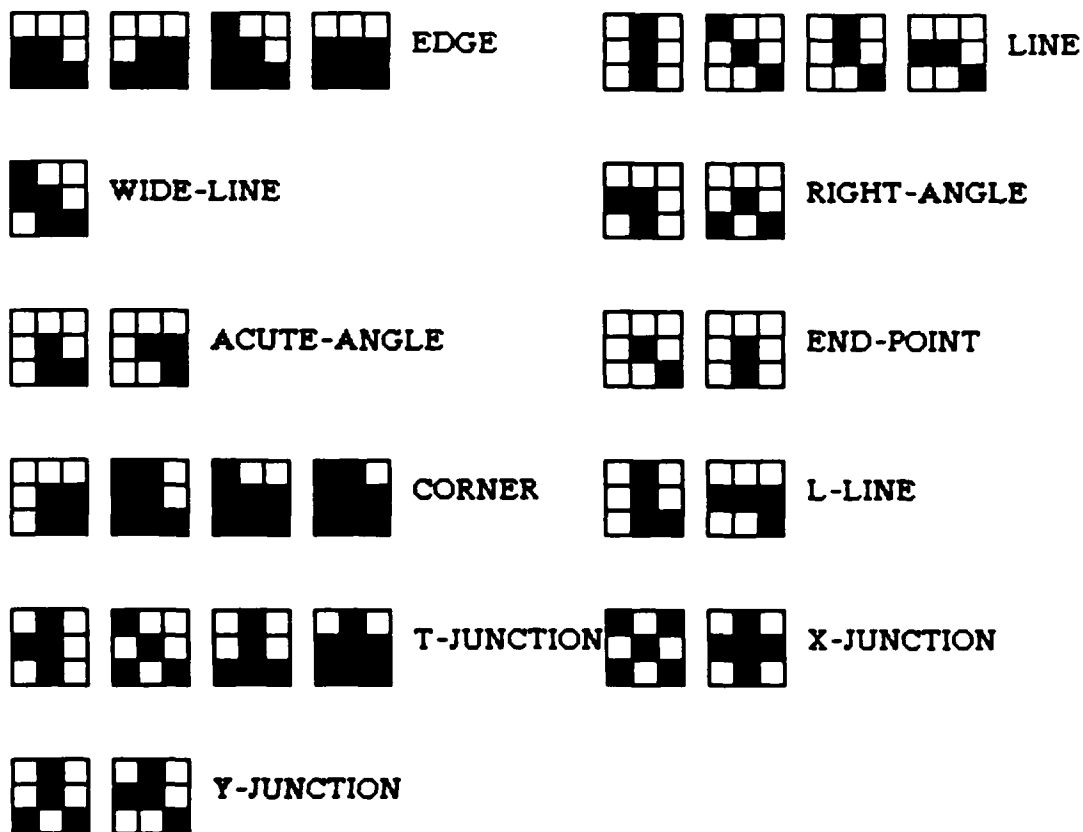


Figure 2. The dark CP linear feature masks.

Each mask defines a partition of the 3×3 neighborhood of the CP into dark and bright pixels. If the CP is dark, we will call the set of dark pixels the *foreground* and the set of bright pixels the *background*.

Table 1 shows the $\{f, 9 - f\}$ partition types for each class of masks, (where f and $9 - f$ denote the numbers of foreground and background pixels respectively) and gives the number of masks of each partition type.

Table 1 - No. of Masks in each Class and Partition Type				
Type No.	Class	Partition Types	No. of Masks	Total
1	Edge	$\{5,4\}, \{6,3\}$	8, 8	16
2	Line	$\{3,6\}$	12	12
3	Wide-Line	$\{5,4\}$	4	4
4	Right-Angle	$\{3,6\}$	8	8
5	Acute-Angle	$\{3,6\}$	8	8
6	End-Point	$\{2,7\}$	8	8
7	Corner	$\{4,5\}, \{7,2\}, \{8,1\}$	4, 8, 4	16
8	L-Line	$\{4,5\}$	8	8
9	T-Junction	$\{4,5\}, \{5,4\}, \{7,2\}$	8, 4, 4	16
10	X-Junction	$\{5,4\}$	2	2
11	Y-Junction	$\{4,5\}$	8	8

Table 1 counts dark or bright CP masks only; to count both, the numbers should

be doubled. The total number of masks is $2 \cdot 106 = 212$.

In a given 3×3 pixel neighborhood, let the nine pixel gray levels be $z_1 \leq \dots \leq z_9$. Depending on how many of the z 's are equal, there can be up to 8 partitions of the nine pixels into k dark pixels and $9 - k$ bright pixels, where $0 < k < 9$, obtained by choosing a threshold between z_k and z_{k+1} (if they are not equal). Thus any pixel neighborhood can match up to 8 masks (not all of which, however, will be linear feature masks).

2.3. The Confidence Measures

We shall use the following "confidence measures" to characterize the match of a given pixel neighborhood with a given mask:

- (1) *Robustness*, defined by the width of the thresholding interval, i.e. the difference between the minimum gray level of a bright pixel and the maximum gray level value of a dark pixel. Note that robustness > 0 is a necessary precondition for mask matching to occur.
- (2) *Contrast*, defined as the difference between the average gray levels of the dark and bright pixels. [A more informative measure can be obtained by regarding the background region as consisting of connected components; for example, in line masks there are two background components. In this situation we can consider separately the difference between the average gray level of the foreground pixels and the pixels belonging to each background component.]

- (3) *Fisher distance* (see [1], Section 10.2.3), defined by $|\mu_1 - \mu_2|/(\sigma_1^2 + \sigma_2^2)$, where μ_1, μ_2 are the average gray levels in the foreground and background, and σ_1, σ_2 are the corresponding standard deviations. In Section 4 we explain on what basis this measure was applied to *all* the matched masks even though in principle it should have been used only for single background component masks (edge type masks, for instance).
- (4) *Homogeneity* of the foreground and each of the background connected components. "X-homogeneity" (where X stands for any connected component of pixels) may be defined as $1 - \text{var}(X) / \text{var}_{\max}(X)$ where $\text{var}(X)$ and $\text{var}_{\max}(X)$ denote the gray level variance and the maximum possible gray level variance for the X-set of pixels, respectively. Assuming the gray level in the image takes on values between 0 and glv_{\max} , it can easily be shown that

$$\text{var}_{\max}(X) = \begin{cases} 1/4 \text{ } glv_{\max}^2 & ||X|| \text{ even} \\ \frac{||X||^2 - 1}{||X||^2} 1/4 \text{ } glv_{\max}^2 & \text{otherwise} \end{cases}$$

where $||X||$ denotes the cardinality of the X-set of pixels.

- (5) *Foreground road similarity*, defined by the negative of the distance between the average foreground gray level and a given "expected gray level" interval.

The measures have been defined here for single-band images, but they can easily be extended to multi-band images.

3. Data Structures and Algorithm

For several reasons, including ease of modifiability and the availability of a diverse software environment, Lisp was the programming language chosen for the implementation of the proposed system. The Lisp-based data structures used and the algorithm are described in the following subsections.

3.1. Data Types and Structures

Using the extended Franz Lisp environment at the University of Maryland [3], we define an image as an abstract data type (or Lisp "*flavor*") with the following "*instance variables*" or characteristics:

Width, *height*, *number_of_bands*, *band_descriptions*, *data_type*, *ranges*, *row_scale*, *col_scale* and *data*.

Band_description is a list of character strings giving a short description of the contents of the band, one for each of the *number_of_bands* bands.

Data_type describes the type of data in the image arrays (e.g., eight bit integer, list, etc.). All bands have the same type.

Ranges is a list of lists that specify the range of data in each band. For a gray level image, each element of the *ranges* list is a list of two fixnums indicating the extrema of the corresponding band (i.e., the min and max gray level value). For a label image, each element is a list of the possible labels occurring in that band.

Row_scale and *col_scale* are the numbers of meters per pixel in the row and column dimensions, respectively. The scale factors are flonums that are negative if unknown.

Data is the three dimensional array (band row column) of data. The first row of data is the uppermost row of the image.

One of the operations that can be performed on an image (i.e., one of the image flavor *methods*) is extracting a 3×3 neighborhood centered around a point at *row, col.*: (*get_3 \times 3_neighborhood row col.*) The value returned is an ordered list (*center 0 1 2 3 4 5 6 7*) of nine elements, where each element is bound to a sublist consisting of the gray level values of the corresponding numbered pixel (See Fig. 1a), in the different bands of the image.

A look-up table for the labels assigned to all possible binary masks is stored in a 512-entry *mask_set* array, where each element is bound either to one of the mask classes or to nil, depending on the corresponding entry (or *mask_number*).

The desired output of the mask matching procedure is an image (*out_image* instance) of the same size as the input image in which each non-border pixel is labeled. For every labeled pixel, the following Lisp structure is stored:

```
((label (mask_number robustness (min_threshold) contrast
      foreground_homogeneity min_background_homogeneity
      fisher_distance foreground_road_similarity)
  ...)
```

Note that several labels may be associated with each labeled pixel.

3.2. Algorithm Outline

To better understand the algorithm, let us first explain the mask matching procedure in somewhat more detail. As indicated earlier, matching occurs when there exists a threshold dividing the pixels of the 3×3 neighborhood into two groups, dark and bright, such that the pattern of dark and bright is the same as one of the masks.

To find such thresholds, we sort the 9 pixel gray levels in the neighborhood. These 9 sorted values define up to 8 intervals where a threshold can be placed to separate the neighborhood into two non-empty groups of pixels, the foreground and the background. If two or more pixels in the neighborhood have the same gray level, there will be fewer than 8 intervals in which to place the threshold. (Robustness must be strictly positive.) Each threshold may give rise to a match, and thus each pixel may receive several labels.

To allow easy modification while experimenting with various data samples, the following control parameters (in addition to the *input_image* instance and the 512-entry *mask_set* of labels) are provided:

- *Sub_image* to examine, namely a list of the following form: (*x0 y0 ncols nrows*), where (*x0, y0*) denotes the upper left pixel coordinates of the *sub_image*, and *ncols, nrows* are the number of columns and rows. The *sub_image* passed to the algorithm should not include border pixels of the *input_image*. Thus if the *input_image* size is $N \times N$, the *sub_image* and the resulting label image should never exceed $(N - 2) \times (N - 2)$.

- Minimum acceptable robustness for any given threshold. The default value for this parameter was chosen as zero. However, higher values should help eliminate "weakly matched" masks.
- Expected road gray level interval.

The algorithm can now be outlined as follows:

(0) **[Initialization]**

Set $xn \leftarrow (x0 + ncols)$, $yn \leftarrow (y0 + nrows)$, $x \leftarrow x0$, $y \leftarrow y0$, $t \leftarrow 0$ and $label \leftarrow nil$, $mask_info \leftarrow nil$.

(1) **[Read 3×3 Neighborhood]**

Read the gray levels of the 3×3 neighborhood centered at $(x\ y)$.

Set $n \leftarrow (c\ n0\ n1\ n2\ n3\ n4\ n5\ n6\ n7)$.

(2) **[Sort the Neighborhood Gray Levels]**

Sort the n list in increasing order.

Set $sn \leftarrow (sn0\ sn1\ sn2\ sn3\ sn4\ sn5\ sn6\ sn7\ sn8)$.

(3) **[Threshold]**

If $snt < sn(t+1)$, partition the 3×3 neighborhood with respect to the t^{th} threshold, assigning binary coefficients to each of the 9 pixels (for $i=0,1,\dots,8$ $b_i \leftarrow 0$ if $glv(p_i) \leq snt$, otherwise $b_i \leftarrow 1$).

Otherwise, goto step (6).

(4) **[Evaluate Mask No. and Reference Array]**

Evaluate the corresponding $mask_number$.

Reference the *mask_set* array; if *label(mask_number)=nil*, goto step (6).

Otherwise, set *label* \leftarrow *mask_set*[*mask_number*].

(5) [Collect Information]

Compute all confidence measures associated with the matched masks, namely robustness, contrast, foreground/background homogeneities, Fisher distance, etc.

Insert the above information into the *mask_info* list associated with the (*x*, *y*) element of the *out_image* array.

(6) [Increment Threshold]

Set *t* \leftarrow *t* + 1; if *t* < 8, return to step (3). Otherwise, set *t* \leftarrow 0.

(7) [Scan the Image]

Set *y* \leftarrow *y* + 1; if *y* \leq *yn*, return to step (1).

Otherwise, set *y* \leftarrow *y* 0, *x* \leftarrow *x* + 1. If *x* \leq *xn*, return to step (1).

(8) [Terminate]

An output example, generated by the algorithm for a 10×10 *sub_image*, is illustrated in the following figures. Figures 3a and 3b show the bright CP masks in the corresponding 8×8 sub-image and (respectively) the robustness and contrast values associated with each mask. (The number at the center of each pixel denotes its gray level value.)

In [2], criteria for the consistency of pairs of masks centered at adjacent pixels are defined. Figure 3c shows the consistent pairs of masks in Figures 3a-b, linked by lines. By tracing along the links, we see that the image contains a

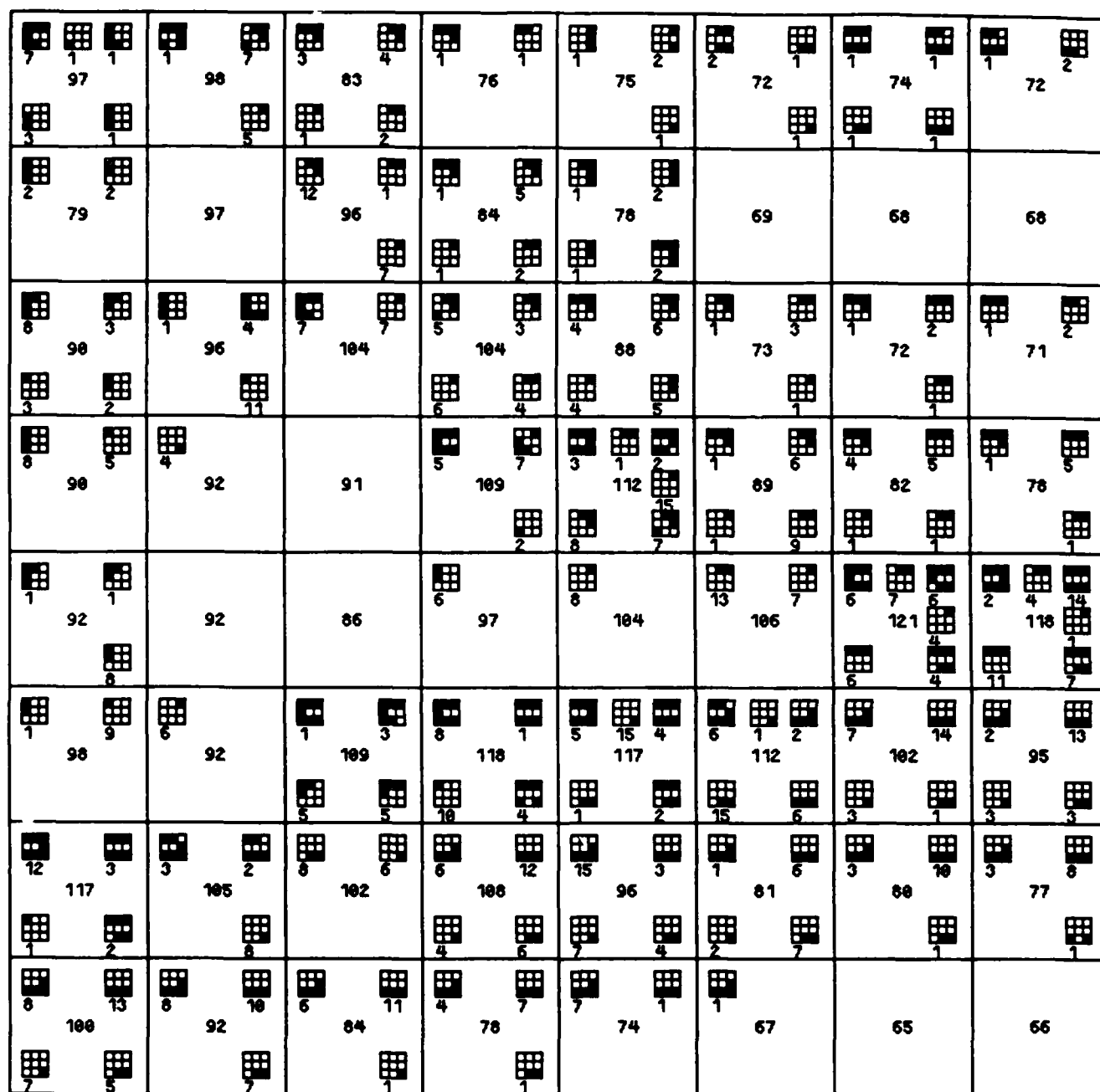


Figure 3a. An example of 8×8 bright CP mask output with associated robustness values.

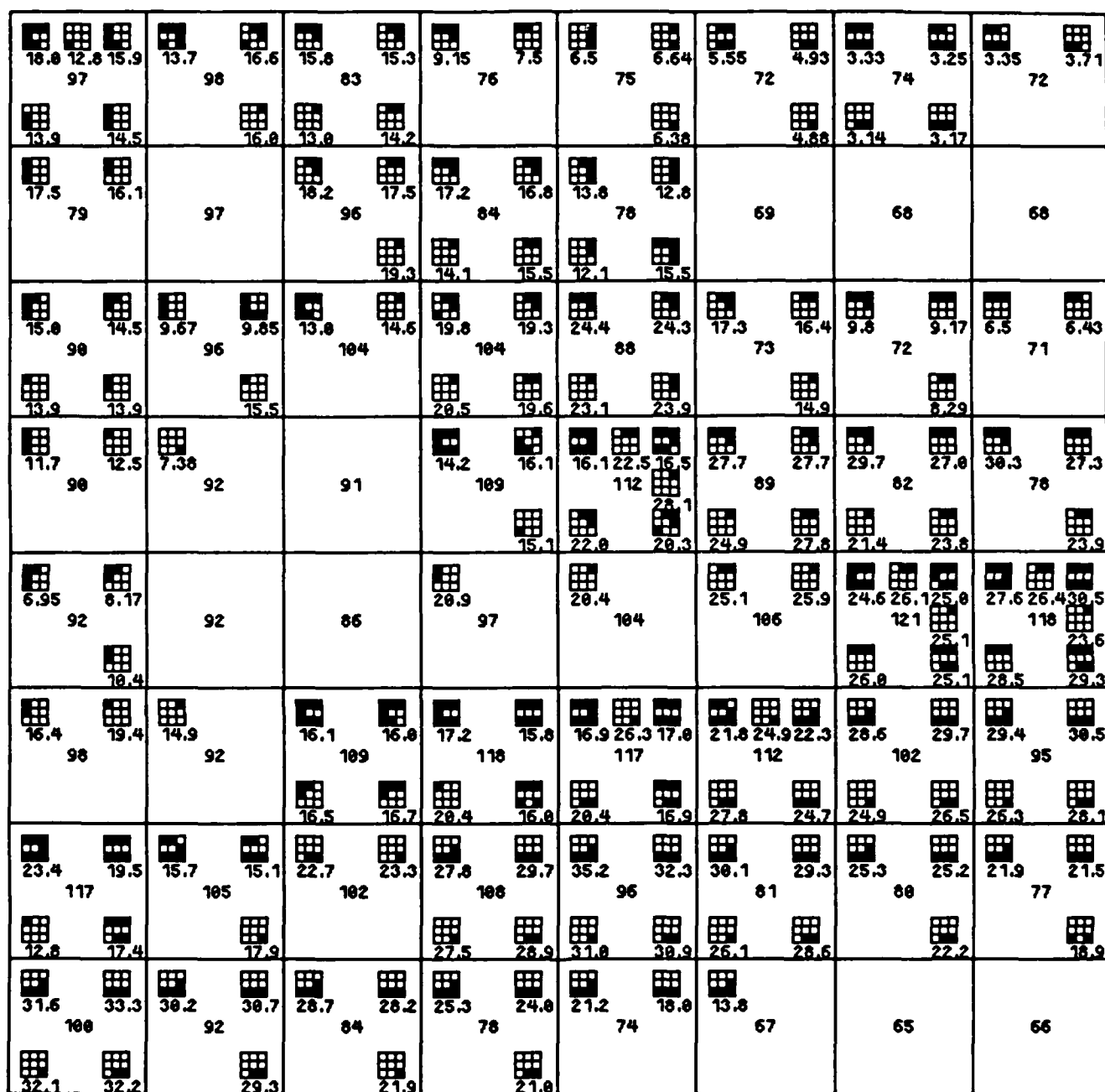


Figure 3b. An example of 8×8 bright CP mask output with associated contrast values.

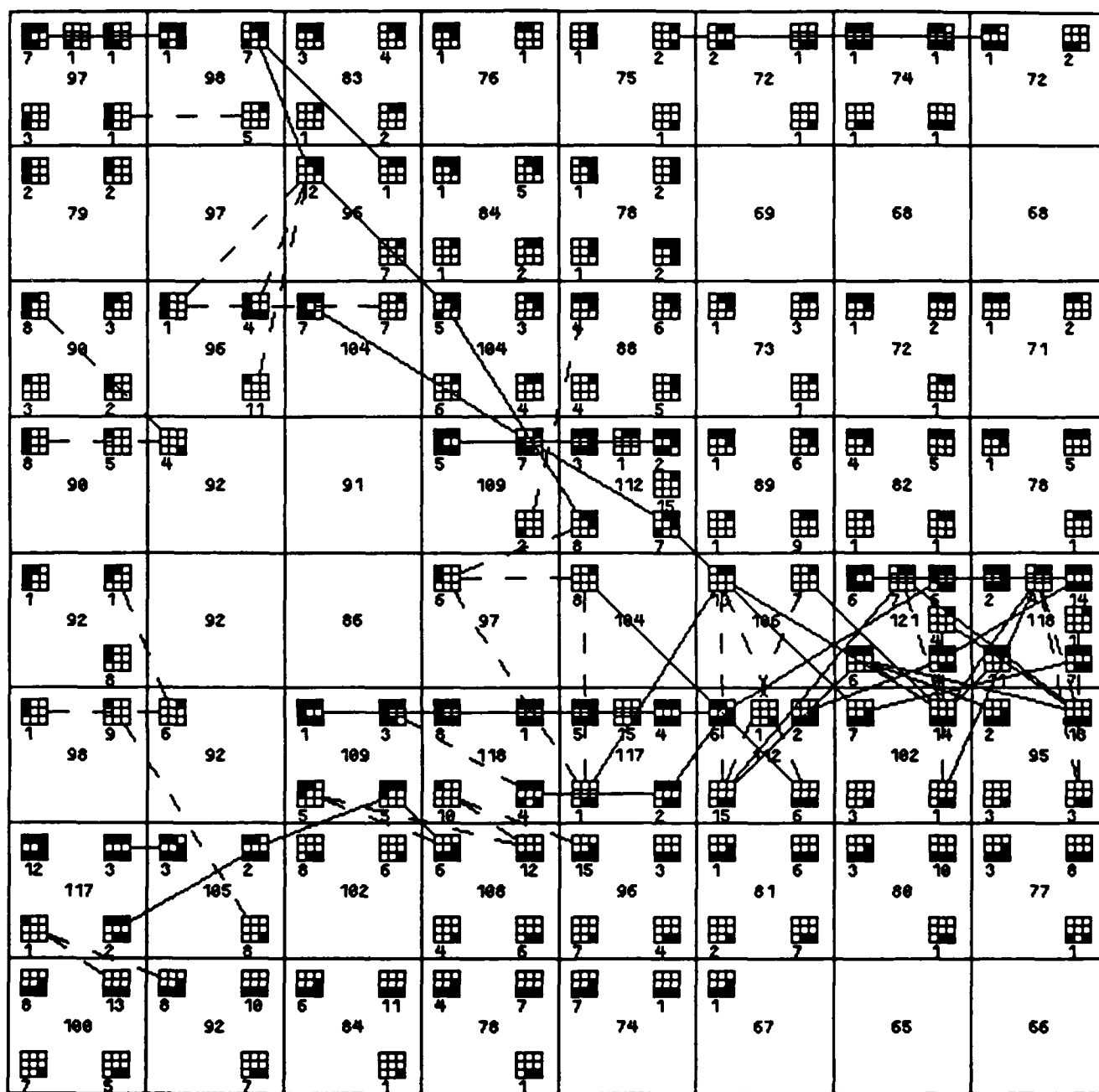


Figure 3c. Consistency links found for the bright CP masks.

branching linear feature, with one branch, roughly horizontal, just below the center of the image, and the other, roughly diagonal, extending upward and leftward from the center.

3.3. Suppressing "Weak" Masks

One might think that it is not necessary to make use of all the masks that match the neighborhood of each pixel, but only of the "strongest" ones, say those having the highest robustness or contrast values. A simple experiment shows, however, that this is not true. Figures 4-8 show, respectively, the consistency links obtained when we keep, at each pixel, only the mask(s) having

- (1) Highest robustness value.
- (2) Two highest robustness values.
- (3) Highest contrast value.
- (4) Two highest contrast values.
- (5) Highest robustness or highest contrast.

(In cases of tied values, all the masks involved are included.) We see that when this is done, the results are very poor, especially for methods (1), (3), and (5). Note that the results obtained using methods (1), (3), and (5) are very similar, indicating that generally, a mask having the highest robustness value at a pixel also has the highest contrast value, and vice versa.

Even the somewhat better results obtained using methods (2) and (4) show that elimination of masks leads to loss of information. (See Figures 5 and 7.)

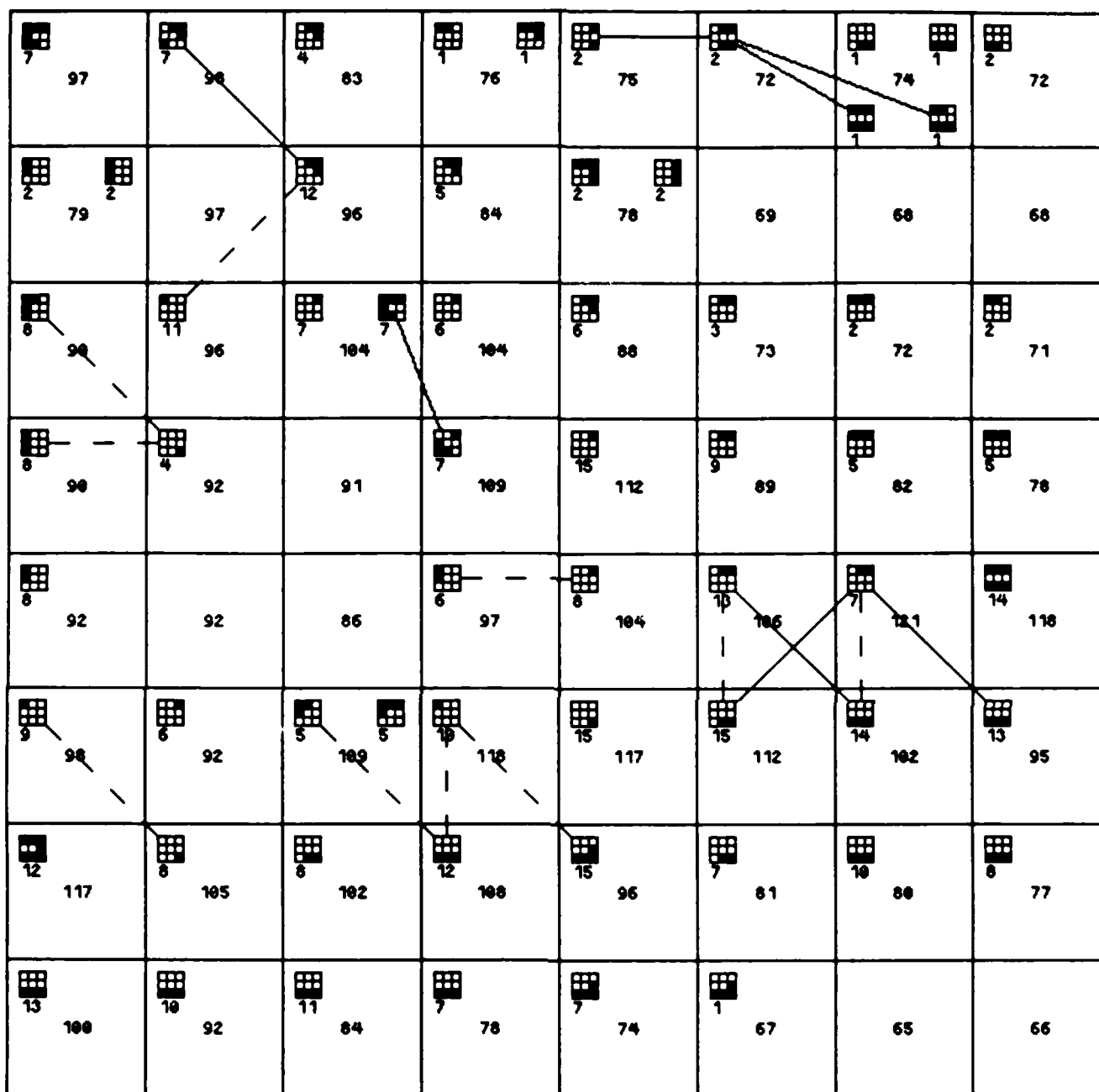


Figure 4. Consistency links found for bright CP masks having the highest robustness values.

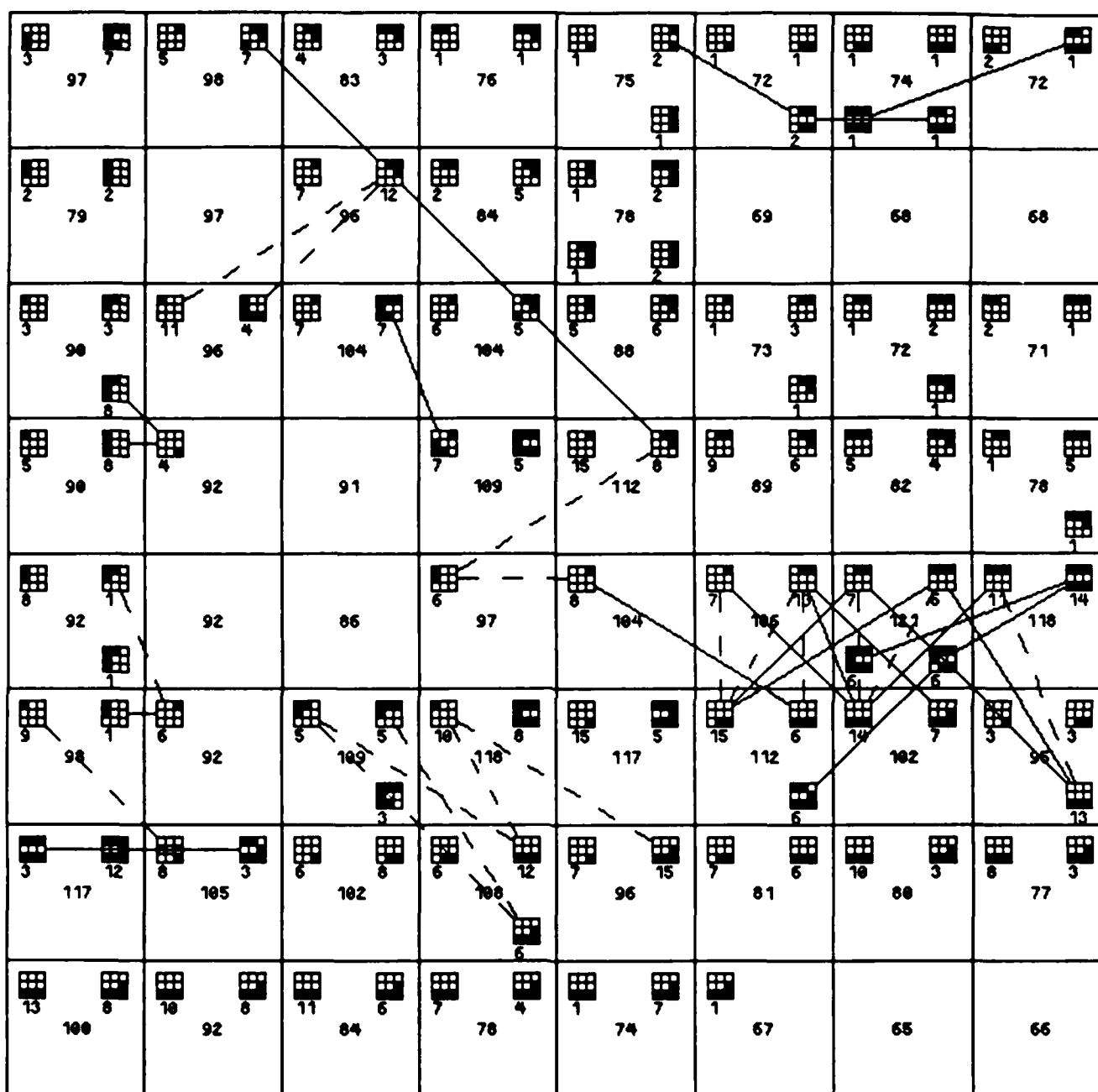


Figure 5. Consistency links found for bright CP masks having the two highest robustness values.
























































 16.0 97	 16.6 98	 15.8 83	 9.15 76	 6.64 75	 5.55 72	 3.33 74	 3.71 72
 17.5 79	97	 19.3 96	 17.2 84	 15.5 78	69	68	68
 15.8 90	 15.5 96	 14.6 104	 20.5 104	 24.4 88	 17.3 73	 9.8 72	 6.5 71
 12.5 90	 7.38 92	91	 16.1 109	 26.1 112	 27.8 89	 29.7 82	 38.3 78
 10.4 92	92	86	 20.9 97	 20.4 104	 25.3 105	 25.1 121	 30.5 116
 19.4 98	 14.9 92	 16.7 109	 29.4 110	 26.3 117	 27.8 112	 29.7 102	 30.5 95
 23.4 117	 17.9 105	 23.3 102	 29.7 108	 35.2 96	 30.1 81	 25.3 80	 21.9 77
 33.3 100	 30.7 92	 28.7 84	 25.3 78	 21.2 74	 13.8 67	65	66

Figure 6. Consistency links found for bright CP masks having the highest contrast values.

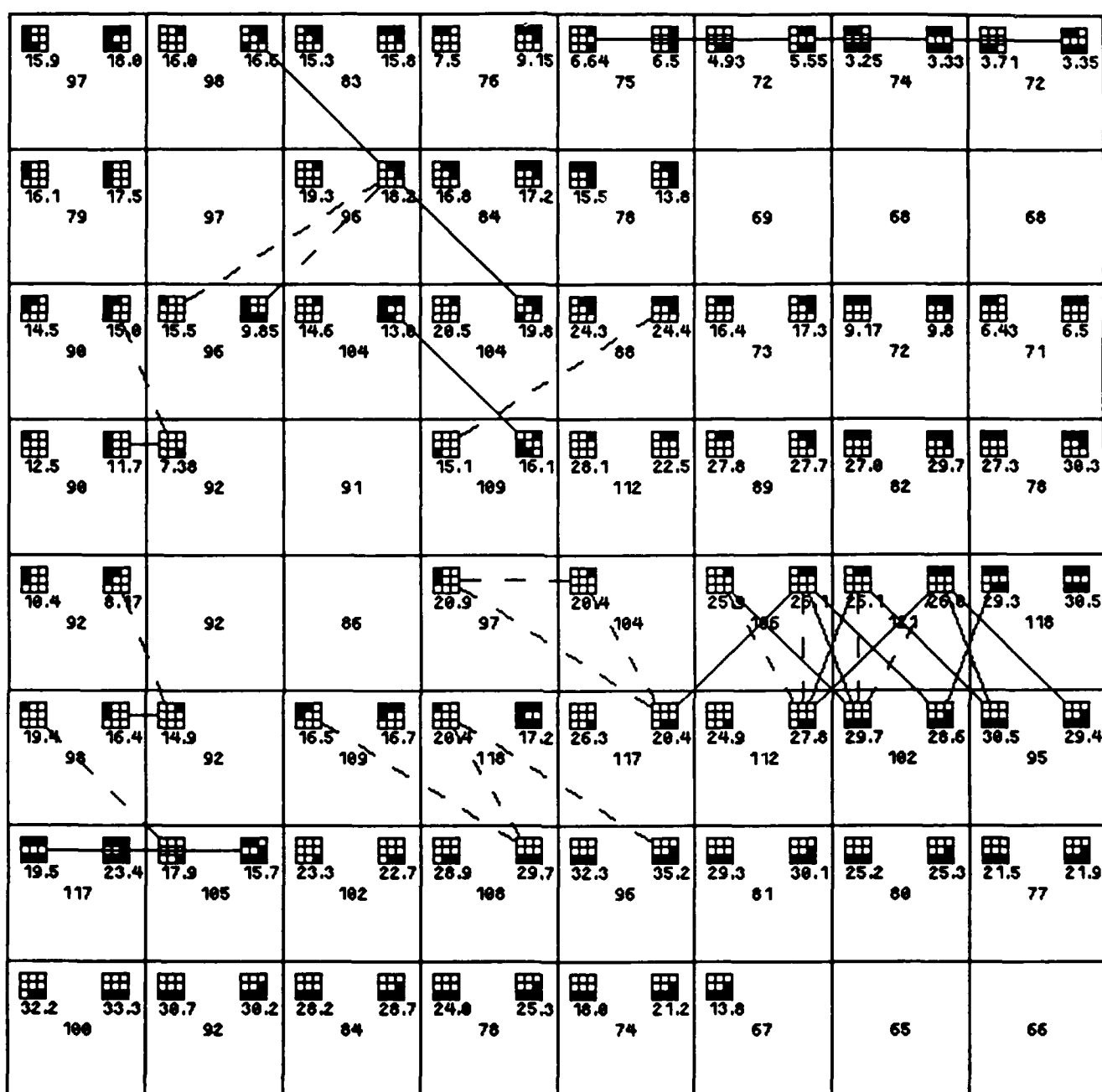


Figure 7. Consistency links found for bright CP masks having the two highest contrast values.

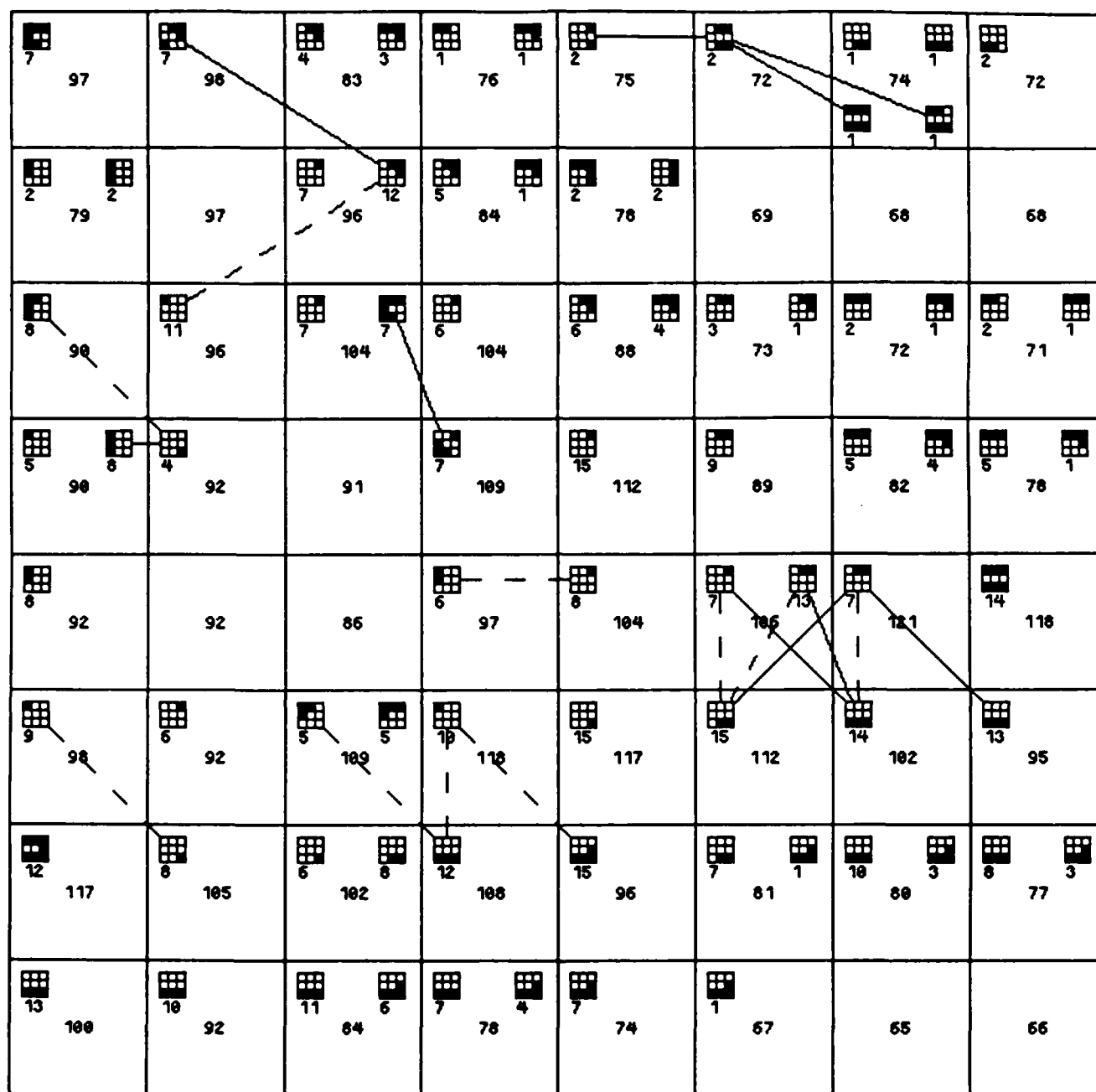


Figure 8. Consistency links found for bright CP masks having either highest robustness or highest contrast. (Robustness values are associated with each mask.)

Thus, it is not desirable to suppress any masks at this stage, in order to maintain all available information at the pixel level for use in the linear feature extraction process.

4. Statistical Studies

It is of interest to study statistical properties of the confidence measures and other parameters associated with the mask matching process. The purpose of these studies is to investigate to what extent it would be useful to make use of statistical properties in the linear feature detection process. The studies performed treat the following issues:

- Evaluation of the frequency of occurrence of classes of matched masks in a given image.
- Computation of the distribution of the various confidence measures associated with the matched masks (e.g., robustness, contrast, Fisher distance, etc.), and of the corresponding statistical parameters (i.e., the maximum, mean, median, and variance values), as functions of the class of matched masks.
- Comparison between the statistical results obtained for the dark CP masks and those for the bright CP masks, as a function of the class of matched masks.
- Comparison between confidence measures evaluated for matched masks at pixels where a non-linear line detector (described in Section 10.3.2 of [1] and referred to as NLD hereafter) responds, and the same measures evaluated at

pixels where the above non-linear detector does not respond.

In repeating the above studies for various image instances, we want to find out whether or not there exists a correlation between the experimental results obtained and the presence of an extensive road network in a given image. For example, it would be of interest to establish that the higher the frequency of occurrence of line-like masks in a certain image is, the more likely road segments are to be found in that image.

4.1. Classification of Image Types

The statistical results given in the following subsections were obtained for the sub-images shown in Figures 9a-b and for 64×64 "white noise" images generated using [4] (for $\mu=127$, and $\sigma=5, 10, 15, 20, 25, 30, 35$). Figures 9a-b show 128×128 and 256×475 gray level images respectively, with four selected window overlays in each (denoted by w1, w2, ..., w8.). The image of Figure 9a (called "Greenbelt1") is a sub-image of the image shown in Figure 9b (called "Greenbelt"), which was digitized from an aerial photograph of the Greenbelt, Maryland area. The quadrants of Figure 9a will be referred to below as "upper-left", "upper-right", etc.

In order to relate the statistical properties to the presence of roads in an image, we have classified the sub-images into the following three categories:

- "suburban" and "highway" images - e.g., windows 4, 7, and 8.
- "homogeneous" images - e.g., windows 1, 2, 3, 5, and 6.

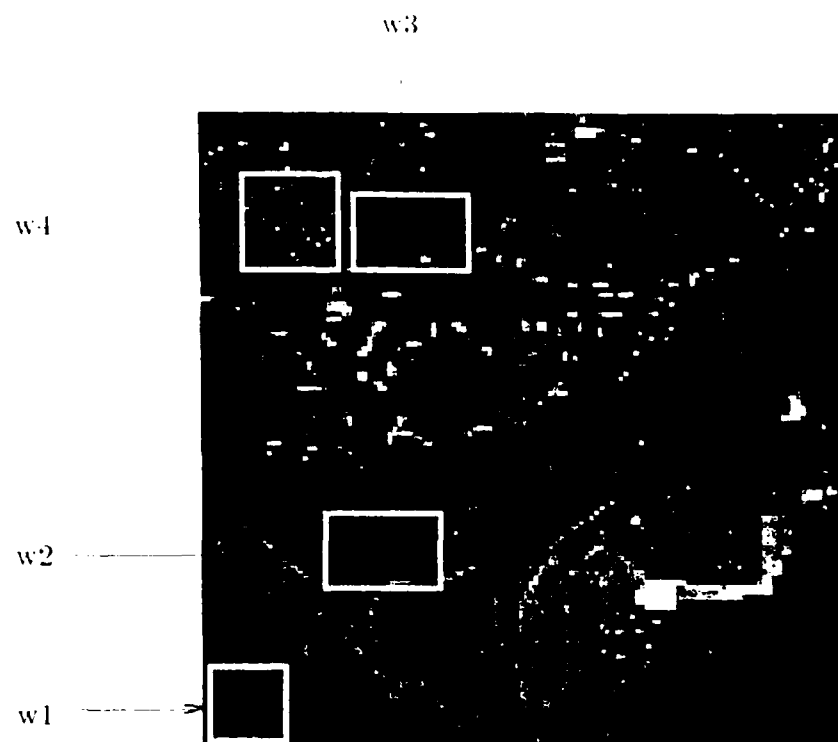


Figure 9a. "Greenbelt1" Image (128×128)
with four selected windows.

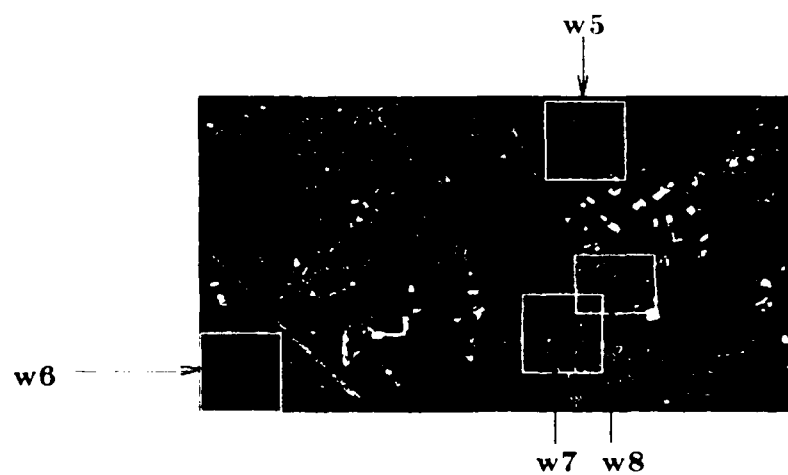


Figure 9b. "Greenbelt" Image (256×475)
with four selected windows.

- “white noise” images.

Figure 10 gives the gray level histograms of several 64×64 sub-images of the Greenbelt area images and of two white noise images. Notice the general resemblance of the histograms of images of the same class. We did not attempt, however, to develop a method of automatically classifying the sub-images by analyzing their histograms.

4.2. Statistical Results

4.2.1. Frequency of Occurrence of Classes of Masks

The histograms of frequencies of occurrence of the classes of bright CP masks (in the images referred to in Figure 10) are shown in Figure 11. Again, the distributions obtained for images of the same type are characteristic. Moreover, the common characteristics of this distribution for images of different types (e.g., the common general shape and the maximal frequency obtained for Corner masks) should be noticed. However, the frequencies of occurrence of Edge and Corner masks are significantly lower in the homogeneous and white noise images, compared with the suburban images. (See Tables 3a-b through 10a-b in the next subsection for the exact figures.)

In order to try to differentiate between the various image types in terms more closely related to the presence of roads in a given image, we apply the following test. Let the set of Line, Wide-Line, and L-Line masks be called “line-like” masks. Given 1-2 pixel wide road images, it is of interest to verify, for example, whether the frequency of occurrence of line-like masks increases in

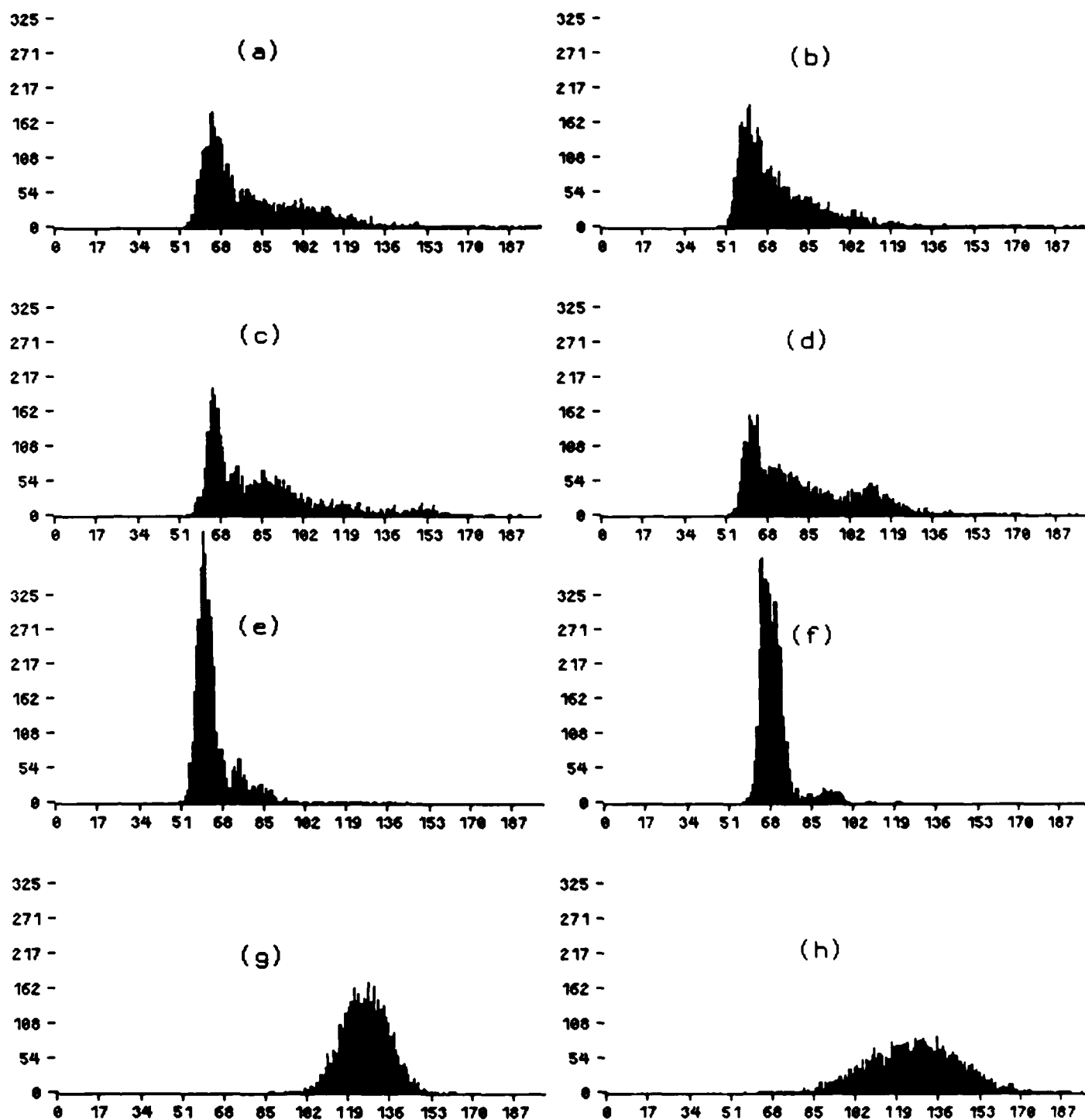


Figure 10. Gray Level Histograms for 64×64 Images.

(a), (b), (c), (d): **suburban**: upper-left, upper-right, lower-left, window_7.

(e), (f): **homogeneous**: window_5, window_6.

(g), (h): **white noise** with $\sigma=10$, $\sigma=20$.

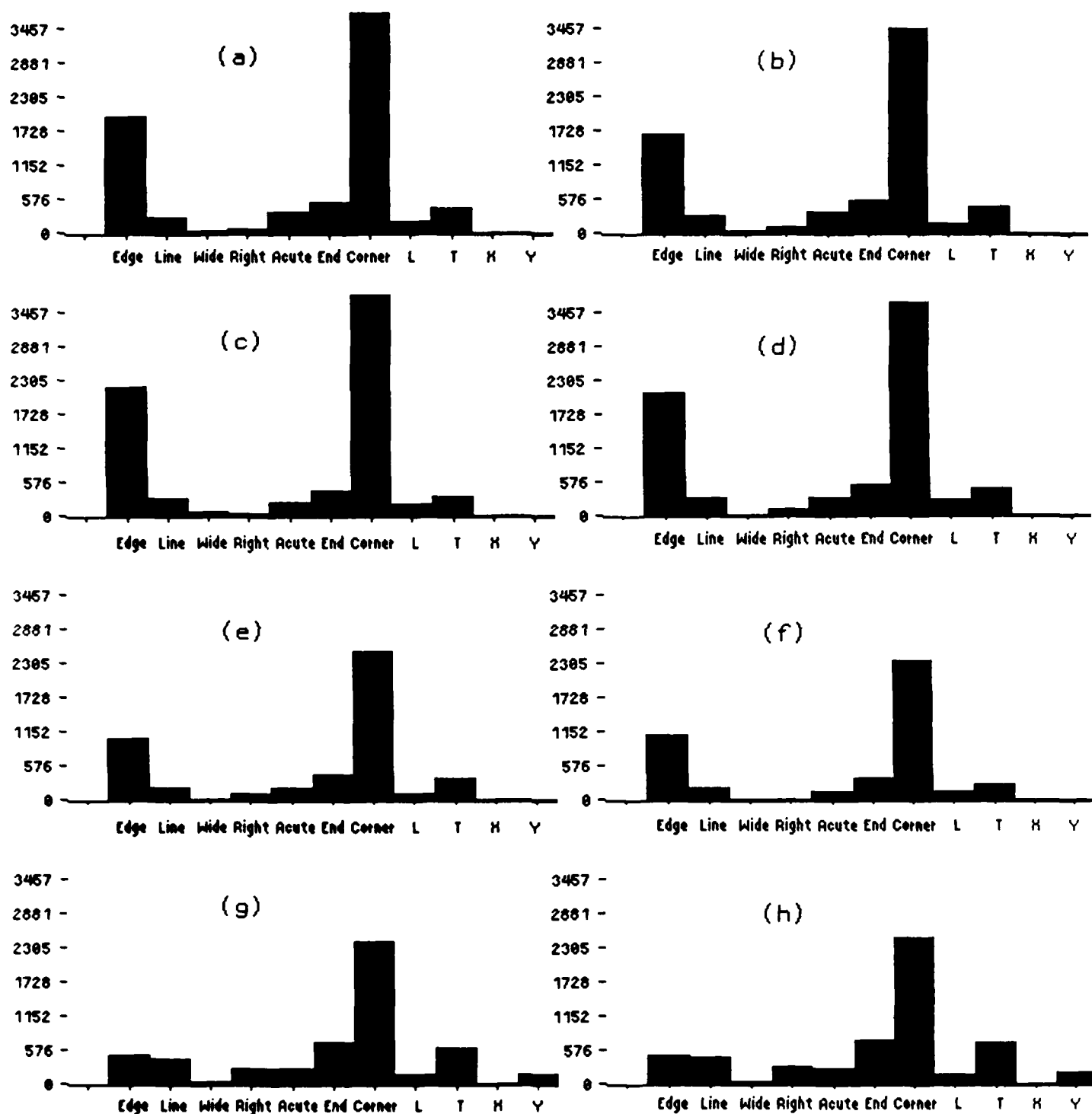


Figure 11. Bright CP Mask Histograms for 64×64 Images.

(a), (b), (c), (d): **suburban**: upper-left, upper-right, lower-left, window_7.

(e), (f): **homogeneous**: window_5, window_6.

(g), (h): **white noise** with $\sigma=10$, $\sigma=20$.

Table 2 - Frequency of Occurrence of Line-Like Masks in Various Images						
Name	Type	Size	% of Dark CP Line-Like Masks		% of Bright CP Line-Like Masks	
			1 Mask per Pixel	Total	1 Mask per Pixel	Total
window_4	suburban	20×20	18.5	24.0	18.8	26.2
window_8	suburban	48×64	11.4	15.1	13.9	18.5
window_7	suburban	64×64	12.5	16.3	14.4	19.7
upper-left	suburban	64×64	13.7	17.3	13.0	16.7
upper-right	suburban	64×64	13.8	17.2	13.9	17.5
lower-left	suburban	64×64	12.5	15.2	14.1	18.1
lower-right	suburban	64×64	12.1	15.2	12.5	16.1
window_1	homogeneous	16×16	13.2	14.3	10.7	10.7
window_2	homogeneous	16×24	14.6	16.2	11.7	14.0
window_3	homogeneous	16×24	10.4	11.0	10.1	12.3
window_5	homogeneous	64×64	11.6	13.7	10.5	11.8
window_6	homogeneous	64×64	12.1	14.2	10.6	12.3
$\mu=127, \sigma=5$	white noise	64×64	14.7	17.2	15.2	17.4
$\mu=127, \sigma=10$	white noise	64×64	17.7	20.7	17.8	20.8
$\mu=127, \sigma=15$	white noise	64×64	17.9	20.6	17.2	20.6
$\mu=127, \sigma=20$	white noise	64×64	19.1	22.8	18.2	21.9
$\mu=127, \sigma=25$	white noise	64×64	18.3	22.2	17.8	21.6
$\mu=127, \sigma=30$	white noise	64×64	17.9	21.2	18.7	22.7
$\mu=127, \sigma=35$	white noise	64×64	19.1	23.1	19.7	23.5

suburban images relative to images of other types. The results, summarized in Table 2, reveal that this frequency in fact decreases in the suburban images examined, compared with most of the white noise images. (A theoretical derivation of the frequency of occurrence of masks in white noise images is given in Section 5.) Hence it is impossible to conclude that a given image region is "suburban" based on its frequency of occurrence of line-like masks.

4.2.2. Distribution of the Confidence Measures

The study performed next computes the distributions of the mask confidence measures (robustness, contrast, and Fisher distance) and their statistical properties (maximum, mean, variance, and median) for the different classes of masks in various types of images. The numerical results obtained for the set of 64×64 images used in the previous subsections are given in Tables 3a-b through 10a-b. (Tables for additional images are appended to this report.) The robustness, contrast, and Fisher distance histograms for the bright CP Line masks are shown in Figures 12-14. Based on these results, the following observations can be made.

- (1) The statistical properties are most similar for images of the same type. (The numerical values obtained for the white noise images differ from one another, but their statistical properties (as functions of the class of masks) are qualitatively similar.)
- (2) The mean and median robustness and contrast values of the matched masks of given classes are generally greater in the suburban images than in the homogeneous images. On the other hand, the mean and median Fisher distance values in the homogeneous images are generally slightly greater than in the suburban images. Also, the Fisher distance variance over all matched masks of a given class is usually smaller in the homogeneous images, except in degenerate cases where the Fisher distance is infinite.
- (3) The values obtained for the statistical properties in the white noise images strongly depend on σ . The values obtained for $\sigma=5$, for example, are similar

to the values for the homogeneous images (see Tables B-7a-b, and Tables 7a-b, 8a-b), whereas the results for $\sigma=10-20$ are much closer to those obtained for the suburban images (see Table 10a-b, Tables 3a-b through 6a-b, and Figure 12). Generally, the frequency of occurrence and the mean and median values of robustness and contrast of given classes of masks in white noise images are increasing functions of σ . (See the theoretical proof of this in Section 5.) The Fisher distance in white noise images seems to vary little, with respect to either σ or the class of masks.

- (4) The contrast histograms of the bright CP Line masks in the white noise images strongly resemble the gray level distributions in the images themselves (see Figures 10, 13). They differ greatly in shape from the contrast histograms obtained for the suburban images; the contrast of the Line masks in the suburban images is more uniformly distributed. The maximal robustness and contrast values for the suburban images are usually greater than in the white noise images. (See also the robustness and contrast histograms of the bright CP Line masks, shown in Figures 12-14.) These observations might suggest the selection of masks based on robustness/contrast criteria, but the consequences of doing this are unsatisfactory, as illustrated in Section 3.3.

These results may be useful in distinguishing between suburban and homogeneous images, but do not seem to be useful in distinguishing between suburban and white noise images.

Table 3a - Statistics for Dark Center Pixel Masks in Upper-left Quadrant of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1976	127.0	7.6	113.9	4.0	141.7	28.4	501.9	22.8	29.7
Line	315	45.0	6.1	72.3	3.0	97.2	22.8	272.5	21.3	9.5
Wide-Line	85	30.0	4.2	26.7	3.0	99.9	23.3	365.4	18.6	4.6
Right-Angle	122	14.0	2.7	7.7	1.5	71.7	17.4	176.7	14.4	3.9
Acute-Angle	355	20.0	3.3	10.5	2.0	74.7	19.4	179.5	17.2	6.8
End-Point	518	48.0	3.6	16.7	2.0	107.1	19.5	199.6	17.1	5.3
Corner	3981	133.0	12.2	246.3	6.0	158.1	31.8	699.1	24.3	47.2
L-Line	268	38.0	4.5	27.3	2.0	85.1	23.8	305.2	21.0	9.4
T-Junction	470	81.0	5.3	59.9	3.0	118.3	22.8	312.8	18.8	10.4
X-Junction	7	3.0	1.6	0.5	1.0	19.4	10.7	33.6	10.2	4.9
Y-Junction	42	19.0	3.4	14.4	2.0	51.7	19.3	170.3	16.7	5.2

Table 3b - Statistics for Bright Center Pixel Masks in Upper-left Quadrant of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	2009	64.0	6.3	57.2	3.0	112.4	27.9	391.3	23.8	13.2
Line	315	67.0	7.6	98.8	3.0	119.7	27.2	535.1	20.7	9.4
Wide-Line	81	50.0	6.9	73.0	4.0	78.9	27.4	397.4	20.8	5.7
Right-Angle	138	34.0	6.3	49.4	3.0	103.2	30.5	437.7	25.3	6.3
Acute-Angle	418	57.0	6.6	74.9	3.0	110.5	30.6	500.6	24.4	10.9
End Point	568	104.0	9.1	148.9	4.0	137.9	31.6	637.9	24.3	13.1
Corner	3769	65.0	5.1	38.6	3.0	115.5	24.1	310.5	20.6	13.8
L-Line	246	48.0	5.4	39.4	3.0	115.7	25.6	426.7	19.7	8.9
T-Junction	482	20.0	3.2	8.8	2.0	92.4	22.2	257.2	19.0	6.5
X-Junction	4	9.0	3.8	9.2	2.0	41.8	23.5	117.7	19.2	2.8
Y-Junction	49	14.0	3.1	9.3	2.0	61.4	19.0	202.6	16.0	4.1

Table 4a - Statistics for Dark Center Pixel Masks in Upper-right Quadrant of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1773	125.0	7.5	123.0	4.0	155.0	28.0	573.7	20.8	28.8
Line	326	55.0	3.2	19.2	2.0	95.5	19.2	287.1	13.4	7.5
Wide-Line	95	17.0	3.4	9.5	2.0	69.8	21.3	250.2	16.8	4.9
Right-Angle	158	20.0	2.4	6.5	2.0	66.0	15.6	151.8	12.4	7.8
Acute-Angle	329	32.0	3.0	11.3	2.0	90.0	19.4	191.5	15.5	6.5
End-Point	521	19.0	2.9	7.6	2.0	86.4	17.0	178.6	13.1	5.8
Corner	3726	161.0	13.1	323.0	6.0	185.4	31.7	844.2	22.0	36.1
L-Line	240	75.0	4.4	45.3	2.0	108.9	24.2	422.2	16.8	9.4
T-Junction	545	69.0	5.4	55.1	3.0	111.6	23.7	427.2	17.5	10.3
X-Junction	6	10.0	2.7	10.9	1.0	45.7	15.2	199.5	10.1	3.7
Y-Junction	55	14.0	2.9	10.8	1.0	62.8	15.8	196.2	9.8	6.2

Table 4b - Statistics for Bright Center Pixel Masks in Upper-right Quadrant of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1725	103.0	5.7	60.0	3.0	140.8	25.6	419.6	19.6	13.1
Line	350	123.0	8.1	209.3	3.0	149.7	27.7	702.2	18.7	22.0
Wide-Line	104	18.0	3.3	12.5	2.0	61.1	18.5	127.7	15.8	5.7
Right-Angle	166	51.0	4.9	37.3	3.0	102.5	26.0	428.5	19.7	6.8
Acute-Angle	412	64.0	5.2	51.4	3.0	118.8	26.8	495.8	19.7	9.0
End-Point	611	111.0	11.5	309.3	4.0	158.4	31.8	815.8	21.0	18.8
Corner	3504	88.0	4.7	36.6	3.0	127.6	21.8	300.2	17.4	18.3
L-Line	220	45.0	4.6	27.1	3.0	114.7	27.3	445.5	22.2	6.7
T-Junction	503	26.0	3.3	12.7	2.0	117.4	22.0	347.5	16.6	7.0
X-Junction	7	9.0	3.3	6.2	2.0	71.6	31.5	326.4	26.3	2.7
Y-Junction	72	20.0	3.6	16.2	2.0	66.4	18.7	203.7	14.8	5.0

Table 5a - Statistics for Dark Center Pixel Masks in Lower-left Quadrant of Greenbelt I

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	2353	78.0	8.0	101.3	4.0	95.5	24.8	273.5	21.7	48.1
Line	294	34.0	3.3	16.5	2.0	56.8	15.0	137.9	11.7	17.7
Wide-Line	96	34.0	5.0	39.3	2.0	60.4	17.9	196.3	12.2	9.3
Right-Angle	93	11.0	2.5	4.8	2.0	45.0	13.5	90.6	11.3	5.3
Acute-Angle	284	30.0	3.1	17.9	2.0	75.5	16.3	146.1	13.0	5.7
End-Point	447	27.0	2.5	7.2	2.0	49.9	13.9	106.5	11.1	6.3
Corner	4014	98.0	7.3	85.2	4.0	113.0	23.3	302.4	19.7	33.5
L-Line	196	21.0	3.4	13.5	2.0	66.7	17.0	182.2	13.2	9.9
T-Junction	345	40.0	3.3	16.1	2.0	88.7	16.2	205.6	12.2	7.9
X-Junction	3	3.0	1.7	0.9	1.0	16.4	9.4	29.9	8.7	3.9
Y-Junction	30	7.0	2.2	3.6	1.0	36.0	10.9	80.6	9.8	6.0

Table 5b - Statistics for Bright Center Pixel Masks in Lower-left Quadrant of Greenbelt I

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	2241	70.0	6.4	60.2	3.0	81.8	23.5	236.1	21.0	20.5
Line	338	74.0	7.6	113.6	3.0	82.0	23.1	258.6	21.3	24.1
Wide-Line	115	20.0	4.6	13.3	3.0	59.5	19.8	111.0	18.7	5.7
Right-Angle	97	19.0	3.2	11.2	2.0	65.3	18.2	178.0	15.0	4.9
Acute-Angle	277	48.0	3.9	23.0	2.0	79.7	18.5	193.9	16.2	6.3
End-Point	464	86.0	5.0	57.6	3.0	97.6	20.8	270.2	17.4	11.0
Corner	3792	47.0	4.9	37.4	3.0	75.7	20.3	208.6	17.6	14.2
L-Line	242	48.0	4.8	26.2	3.0	68.8	23.1	213.7	22.6	10.8
T-Junction	387	21.0	2.8	8.6	2.0	61.8	16.8	144.1	15.2	5.8
X-Junction	1	6.0	6.0	0.0	6.0	16.1	16.1	0.0	16.1	3.6
Y-Junction	45	13.0	3.1	6.6	2.0	43.4	15.6	129.2	13.2	6.1

Table 6a - Statistics for Dark Center Pixel Masks in window_7 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	2240	79.0	9.0	103.5	5.0	93.3	28.5	261.4	26.9	37.3
Line	311	25.0	4.1	16.0	3.0	59.3	18.8	113.6	17.5	7.7
Wide-Line	65	32.0	5.2	38.6	3.0	46.6	21.4	142.9	22.5	7.1
Right-Angle	157	21.0	3.3	10.6	2.0	49.7	18.8	132.6	18.5	8.5
Acute-Angle	355	29.0	3.7	16.3	2.0	59.2	20.1	129.6	20.2	5.7
End-Point	510	25.0	3.5	11.6	2.0	50.8	18.4	108.6	18.1	9.2
Corner	3916	87.0	9.0	107.1	5.0	131.0	27.7	310.1	25.3	26.0
L-Line	249	31.0	4.6	23.2	3.0	60.8	0.7	125.2	18.9	8.8
T-Junction	536	34.0	4.3	21.3	3.0	83.1	22.3	196.9	21.3	7.3
X-Junction	7	5.0	2.0	2.0	1.0	31.8	17.4	110.5	12.7	4.3
Y-Junction	33	8.0	2.3	3.3	1.0	41.0	14.2	101.9	11.0	4.9

Table 6b - Statistics for Bright Center Pixel Masks in window_7 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	2159	54.0	7.4	59.9	5.0	75.5	27.7	196.1	27.3	22.2
Line	367	68.0	7.0	74.9	4.0	86.2	25.8	249.5	22.3	10.2
Wide-Line	66	31.0	5.8	38.2	3.0	46.4	22.9	97.4	22.0	7.9
Right-Angle	145	23.0	4.4	21.7	3.0	73.3	24.0	242.1	21.2	5.9
Acute-Angle	364	36.0	5.0	28.3	3.0	71.2	24.6	210.8	23.8	10.9
End-Point	560	51.0	5.6	48.0	3.0	84.9	25.4	271.3	22.5	8.6
Corner	3675	43.0	5.5	33.1	3.0	67.9	24.2	162.7	23.7	12.2
L-Line	326	37.0	5.5	32.9	3.0	77.8	25.6	224.9	23.5	8.3
T-Junction	524	27.0	3.8	15.5	2.0	69.6	21.5	163.4	20.6	8.3
X-Junction	7	5.0	3.0	3.4	3.0	31.8	17.4	44.4	16.6	3.0
Y-Junction	47	12.0	2.6	6.1	2.0	43.3	15.0	90.9	15.3	7.2

Table 7a - Statistics for Dark Center Pixel Masks in window_5 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Med
Edge	986	24.0	2.7	7.0	40.2	8.9	39.1	13.9	3.4	1.7
Line	310	6.0	1.6	1.4	16.2	5.5	12.6	7.5	3.0	0.8
Wide-Line	40	3.0	1.1	0.2	12.8	5.1	8.2	6.1	3.1	0.6
Right-Angle	140	5.0	1.2	0.3	14.7	4.3	7.0	6.0	2.9	0.8
Acute-Angle	190	9.0	1.4	0.9	14.3	4.8	8.2	6.9	3.0	0.8
End-Point	458	7.0	1.6	1.0	16.0	5.1	9.1	7.2	2.7	0.8
Corner	2580	31.0	2.7	9.0	43.1	7.2	32.9	24.0	3.1	2.1
L-Line	178	10.0	1.8	2.6	16.8	6.1	15.0	7.4	3.3	1.2
T-Junction	395	6.0	1.5	0.9	22.8	4.8	9.2	7.2	3.0	0.8
X-Junction	14	2.0	1.1	0.1	7.4	3.4	2.0	4.6	2.9	0.5
Y-Junction	54	4.0	1.2	0.3	11.2	3.7	3.2	4.8	3.1	0.6

Table 7b - Statistics for Bright Center Pixel Masks in window_5 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Med
Edge	1072	18.0	2.7	7.0	36.0	9.0	37.0	∞	∞	3.2
Line	254	8.0	1.7	1.7	20.3	4.9	10.2	∞	∞	3.1
Wide-Line	50	9.0	1.5	1.5	23.5	4.8	13.6	6.6	3.1	0.8
Right-Angle	153	6.0	1.4	0.7	22.0	4.8	10.9	9.8	2.9	1.0
Acute-Angle	241	14.0	1.6	1.7	28.0	5.9	19.1	6.1	3.0	0.8
End-Point	470	20.0	1.7	2.5	29.4	5.2	14.5	10.6	2.9	1.1
Corner	2551	24.0	2.3	5.1	40.4	6.9	26.8	∞	∞	2.6
L-Line	148	8.0	1.3	0.7	14.8	5.2	8.8	7.1	3.2	0.9
T-Junction	411	9.0	1.4	0.7	24.4	4.9	9.7	6.2	2.9	0.8
X-Junction	5	2.0	1.2	0.2	4.1	3.6	0.3	4.4	3.1	0.6
Y-Junction	60	3.0	1.1	0.1	8.0	3.4	2.0	∞	∞	2.9

Table 8a - Statistics for Dark Center Pixel Masks in window_6 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1110	26.0	2.3	6.7	1.0	37.5	6.8	32.1	4.5	∞
Line	305	8.0	1.5	0.8	1.0	16.0	4.1	6.0	3.5	∞
Wide-Line	38	5.0	1.3	0.6	1.0	10.1	4.0	4.5	3.5	5.0
Right-Angle	68	3.0	1.1	0.1	1.0	13.8	3.2	3.8	2.8	6.1
Acute-Angle	175	6.0	1.2	0.4	1.0	19.5	4.0	6.5	3.3	6.5
End-Point	398	7.0	1.3	0.6	1.0	20.7	3.9	5.6	3.4	6.6
Corner	2576	31.0	2.1	5.8	1.0	55.1	5.5	24.6	3.7	∞
L-Line	201	7.0	1.4	0.6	1.0	18.2	4.2	7.3	3.4	∞
T-Junction	310	12.0	1.4	1.3	1.0	24.1	4.3	13.2	3.1	∞
X-Junction	3	1.0	1.0	0.0	1.0	2.1	1.7	0.1	1.7	4.5
Y-Junction	34	5.0	1.3	0.9	1.0	12.2	3.0	5.3	2.3	6.1

Table 8b - Statistics for Bright Center Pixel Masks in window_6 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1143	25.0	2.1	3.8	1.0	39.6	6.4	24.6	4.3	∞
Line	249	23.0	1.9	6.1	1.0	26.0	4.9	20.2	3.3	∞
Wide-Line	42	2.0	1.1	0.1	1.0	18.2	3.9	8.6	3.0	4.1
Right-Angle	72	10.0	1.3	1.4	1.0	22.5	4.1	15.7	3.0	4.5
Acute-Angle	190	10.0	1.2	0.7	1.0	23.2	4.2	11.1	3.1	6.1
End-Point	408	24.0	1.7	4.4	1.0	36.8	4.7	19.3	3.3	∞
Corner	2409	20.0	1.7	1.9	1.0	35.9	4.9	13.5	3.8	∞
L-Line	180	10.0	1.6	1.2	1.0	23.8	5.1	17.1	3.6	9.9
T-Junction	324	7.0	1.3	0.5	1.0	17.6	4.5	11.2	3.1	6.4
X-Junction	8	1.0	1.0	0.0	1.0	3.8	2.3	0.5	2.1	4.4
Y-Junction	27	5.0	1.4	1.4	1.0	11.3	3.2	7.1	2.3	6.1

Table 9a - Statistics for Dark Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=10$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	529	16.0	3.4	6.4	3.0	29.8	15.0	13.5	14.9	8.0
Line	461	15.0	3.3	6.1	3.0	30.2	15.0	15.7	14.8	7.5
Wide-Line	121	11.0	3.3	6.5	2.0	24.8	14.4	12.3	14.4	6.1
Right-Angle	309	15.0	3.3	6.7	2.0	55.3	15.2	21.7	14.5	10.2
Acute-Angle	336	14.0	3.4	6.0	3.0	27.7	15.1	15.8	14.8	8.6
End-Point	754	22.0	4.0	9.5	3.0	75.1	15.7	24.2	15.1	7.5
Corner	2448	28.0	5.0	16.7	4.0	40.0	16.2	24.1	15.6	21.8
L-Line	213	14.0	3.2	6.7	2.0	27.8	15.0	13.0	14.4	10.4
T-Junction	697	17.0	3.6	7.0	3.0	30.6	15.3	15.3	14.9	7.8
X-Junction	47	7.0	2.6	2.2	2.0	21.0	14.1	8.8	14.1	3.9
Y-Junction	198	13.0	3.3	6.5	2.0	29.9	14.8	12.3	14.7	8.6

Table 9b - Statistics for Bright Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=10$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	556	16.0	3.6	6.9	3.0	28.8	15.4	14.9	15.3	8.9
Line	471	16.0	3.6	7.4	3.0	27.0	15.1	14.2	15.0	7.8
Wide-Line	107	9.0	3.0	4.6	2.0	21.6	14.3	11.8	14.3	6.3
Right-Angle	323	14.0	3.3	6.9	3.0	26.5	14.9	14.2	14.5	6.0
Acute-Angle	312	13.0	3.3	6.2	3.0	27.8	15.1	14.1	14.7	12.1
End-Point	755	17.0	3.9	9.1	3.0	30.4	15.4	15.9	15.2	10.2
Corner	2466	115.0	5.2	24.5	4.0	130.4	16.4	32.9	15.7	13.3
L-Line	220	19.0	3.3	7.0	2.0	27.1	15.0	15.3	14.5	11.4
T-Junction	688	16.0	3.6	7.5	3.0	33.5	15.2	17.2	14.8	10.9
X Junction	34	8.0	3.4	4.4	3.0	27.4	15.0	15.2	14.6	4.2
Y-Junction	219	12.0	3.1	6.5	2.0	24.2	14.8	12.5	14.4	5.8

Table 10a - Statistics for Dark Center Pixel Masks in a 64×64 "White Noise" Image with $\mu=127$, $\sigma=20$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	524	33.0	6.5	27.6	5.0	53.3	29.9	70.6	28.9	6.6
Line	524	27.0	6.1	23.9	5.0	56.2	30.3	65.5	30.2	7.4
Wide-Line	108	27.0	5.9	21.9	4.0	54.6	29.5	73.3	28.6	5.8
Right-Angle	339	29.0	6.0	26.2	4.0	58.8	29.8	75.3	28.8	7.4
Acute-Angle	309	27.0	6.6	31.5	5.0	54.8	30.9	75.3	30.2	11.6
End-Point	801	40.0	7.8	40.5	6.0	60.4	31.7	84.9	30.8	9.1
Corner	2574	67.0	9.6	72.0	7.0	90.6	32.3	113.8	30.9	8.3
L-Line	245	28.0	6.2	24.1	5.0	50.9	30.6	64.4	30.3	6.1
T-Junction	743	39.0	6.9	35.6	5.0	65.4	30.8	78.6	30.5	7.3
X-Junction	53	15.0	6.0	15.4	5.0	44.2	27.6	48.2	27.9	5.5
Y-Junction	239	26.0	5.8	21.3	5.0	55.7	29.4	66.7	28.7	5.3

Table 10b - Statistics for Bright Center Pixel Masks in a 64×64 "White Noise" Image with $\mu=127$, $\sigma=20$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	540	29.0	6.4	30.3	5.0	66.2	29.3	72.4	28.4	13.0
Line	517	32.0	6.6	29.4	5.0	65.3	30.6	78.7	30.0	5.5
Wide Line	97	32.0	6.1	28.5	4.0	50.5	29.4	54.8	29.6	5.7
Right Angle	345	34.0	6.5	29.6	5.0	57.3	29.7	76.8	28.5	6.6
Acute-Angle	331	22.0	6.0	18.8	5.0	55.0	29.8	62.0	28.5	5.1
End-Point	803	38.0	7.5	37.3	6.0	68.6	31.3	91.5	30.1	9.4
Corner	2535	104.0	10.2	80.5	8.0	133.3	32.8	118.7	31.8	9.9
L Line	229	26.0	5.8	20.5	4.0	56.6	29.5	62.6	29.2	7.0
T Junction	777	30.0	6.9	31.6	5.0	61.2	30.6	81.3	29.8	6.4
X Junction	60	24.0	5.2	22.6	3.0	54.1	29.5	91.0	27.9	4.4
Y-Junction	247	29.0	6.0	23.5	5.0	53.7	30.1	66.1	29.9	6.5

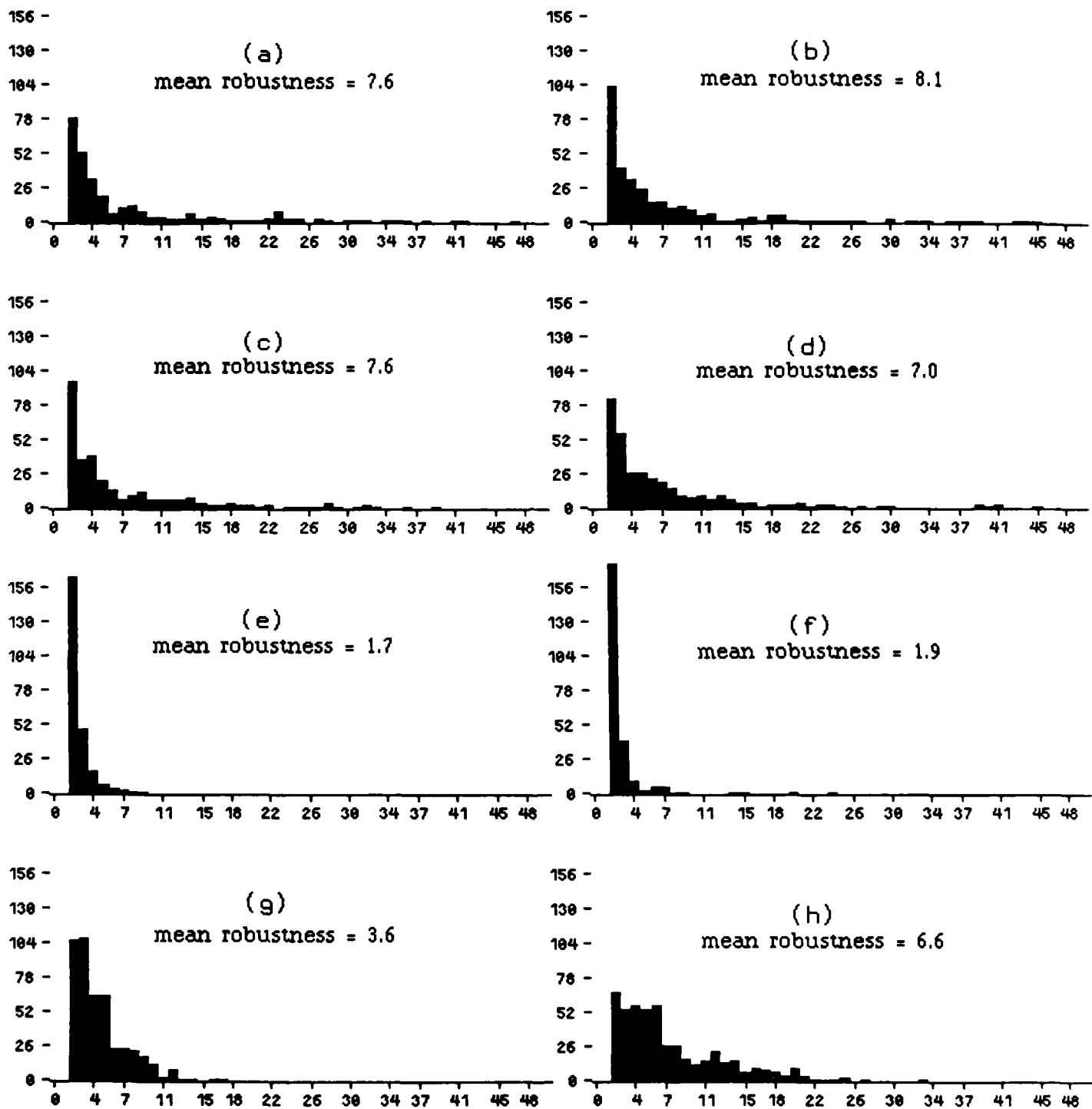


Figure 12. Robustness Histograms for Bright CP Line Masks in 64×64 Images.

(a), (b), (c), (d): **suburban**: upper-left, upper-right, lower-left, window_7.

(e), (f): **homogeneous**: window_5, window_6.

(g), (h): **white noise** with $\sigma=10$, $\sigma=20$.

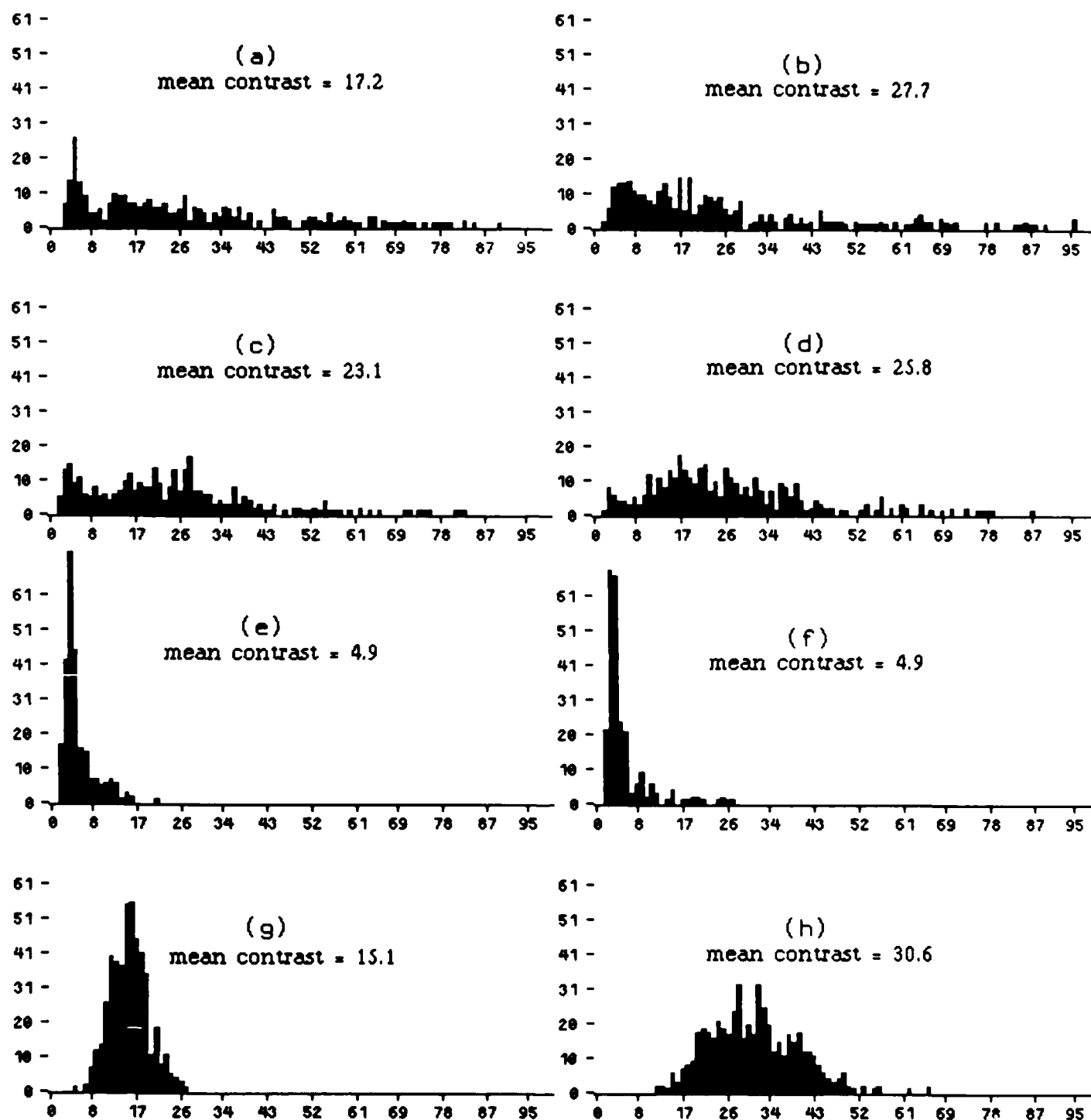


Figure 13. Contrast Histograms for Bright CP Line Masks in 64 x 64 Images.

- (a), (b), (c), (d): **suburban**: upper-left, upper-right, lower-left, window_7.
 (e), (f): **homogeneous**: window_5, window_6.
 (g), (h): **white noise** with $\sigma=10$, $\sigma=20$.

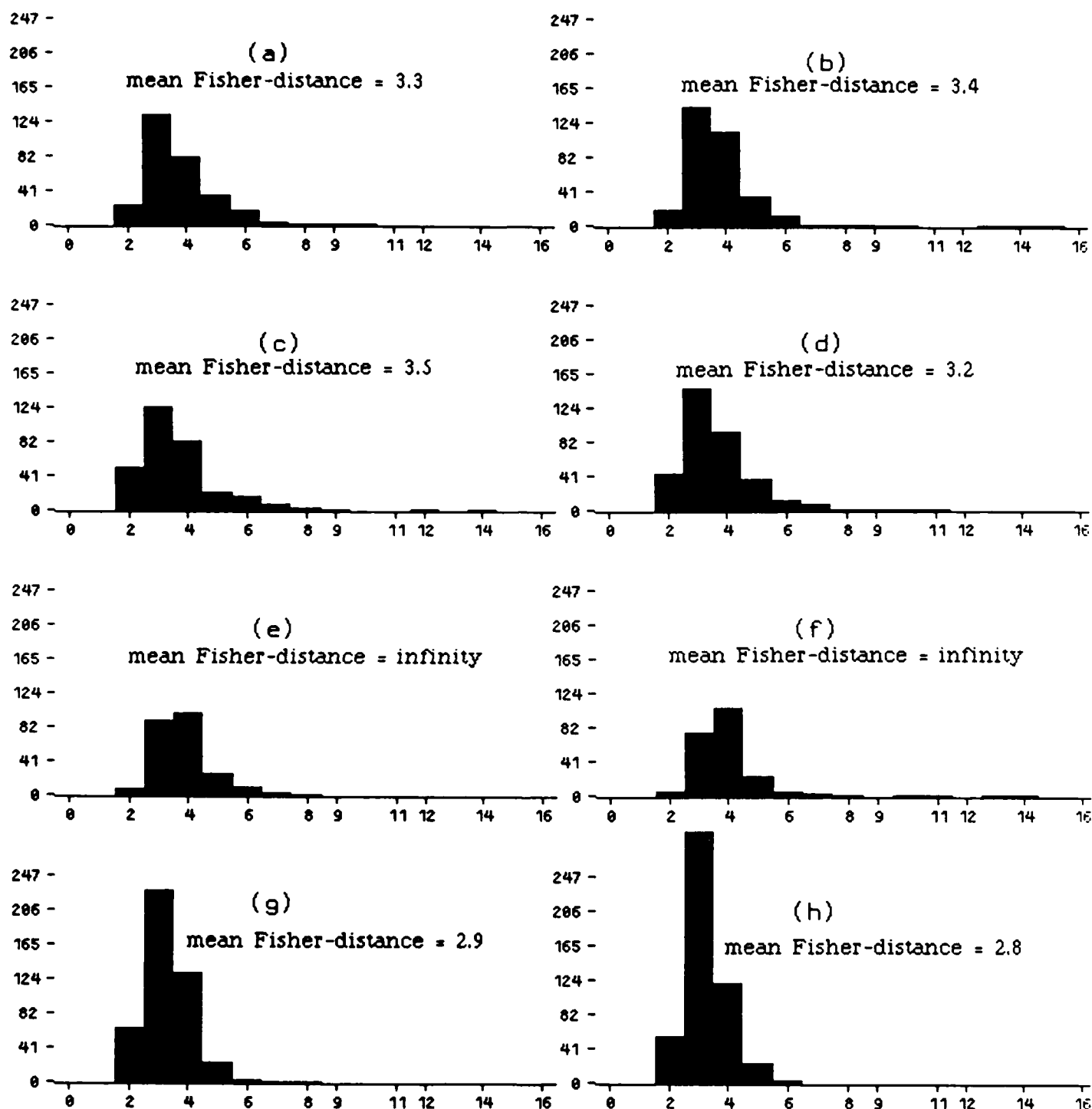


Figure 14. Fisher-Distance Histograms for Bright CP Line Masks in 64×64 Images.

- (a), (b), (c), (d): **suburban**: upper-left, upper-right, lower-left, window_7.
 (e), (f): **homogeneous**: window_5, window_6.
 (g), (h): **white noise** with $\sigma=10$, $\sigma=20$.

4.2.3. Statistics of Dark vs. Bright CP Masks

Linear features in an image include both bright features on a dark background and dark features on a bright background. It is of interest to compare the statistics obtained for bright and dark CP masks, which should respond to these two types of features.

The results presented in Tables 2 through 10a-b indicate the following facts:

- (1) There is no significant difference between the frequencies of occurrence of dark and bright CP masks, even though some of the images seem to contain more bright linear features.
- (2) The values of most of the statistical properties are approximately the same for both dark and bright CP masks in the homogeneous and white noise images.
- (3) In the suburban images, the mean and median robustness and contrast of the bright CP masks are generally greater than those of the dark CP masks. Since the roads in the Greenbelt images are bright, this would suggest that it might be appropriate to suppress weak masks, but we have already seen that this yields poor results. A more interesting suggestion is that if the robustness and contrast values of, say, the bright CP masks are significantly greater than those of the dark CP masks, we have evidence that the image is not homogeneous or white noise, and may contain bright linear features. (This ignores the possibility that an image may contain both dark linear features on a bright background and bright features on a dark background.)

However, even if this assertion is correct, it can only be thought of as an aid in labeling an image region as a whole, but not as a means of detecting individual linear features.

4.2.4. Characterization of "Line-Like" Masks at Pixels where the NLD Responds

If we assume that the nonlinear line detector (NLD) provides "image truth" for the location of linear features, it is of interest to compare the statistics obtained for line-like masks at pixels where the NLD responds with those obtained at pixels where it does not respond.

The results, given in Tables 11-12, and the histograms, shown in Figures 15a-c through 17a-c, indicate the following facts. As might be expected, the frequency of occurrence of bright CP line-like masks at pixels where the NLD responds is generally much greater than at pixels where it does not respond. Similarly, for suburban and homogeneous images, the mean robustness and contrast values at pixels where the NLD responds are greater than the values at pixels where it does not respond. (This is not true for white noise images.) These results show, not surprisingly, that the line-like masks tend to be matched at the same pixels at which the NLD responds.

Table 11 – Statistical Results for Bright CP Line-Like Masks
at Pixels where Nonlinear Detector Responds

Name	Type	Size	% of Masks vs. Magnitude			Mean Robustness vs. Magnitude			Mean Contrast vs. Magnitude			Mean Fisher-distance vs. Magnitude		
			1	5	10	1	5	10	1	5	10	1	5	10
window_4	suburban	20×20	21.9	21.0	20.0	8.4	8.7	9.0	33.0	34.3	35.5	3.2	3.2	3.2
window_8	suburban	48×64	15.8	15.1	13.5	6.1	6.3	6.8	25.5	26.5	28.7	3.2	3.2	3.2
window_7	suburban	64×64	16.2	15.5	14.4	6.8	7.0	7.4	27.1	28.2	29.5	3.3	3.3	3.3
upper-left	suburban	64×64	13.3	11.8	10.0	7.1	7.9	8.9	27.9	31.0	35.0	3.3	3.3	3.3
upper-right	suburban	64×64	13.6	12.9	10.6	7.2	7.5	8.7	28.8	30.3	35.1	3.3	3.3	3.3
lower-left	suburban	64×64	14.2	12.7	11.2	6.6	7.3	7.9	23.7	26.1	28.2	3.5	3.5	3.6
lower-right	suburban	64×64	13.1	12.0	10.4	5.4	5.8	6.4	23.3	25.1	27.8	3.2	3.2	3.2
window_1	homogeneous	16×16	9.2	0	0	1.1	-	-	2.4	-	-	3.7	-	-
window_2	homogeneous	16×24	11.4	7.8	5.2	5.8	7.9	11.0	23.1	32.2	45.2	3.4	3.5	3.8
window_3	homogeneous	16×24	9.7	5.8	3.2	3.1	4.4	6.4	14.5	21.4	32.7	3.1	3.0	3.1
window_5	homogeneous	64×64	8.2	3.0	0.8	1.7	2.5	3.8	5.4	8.6	13.1	3.3	3.3	3.4
window_6	homogeneous	64×64	8.9	2.7	1.1	1.8	3.3	5.2	5.4	10.7	16.3	∞	3.8	4.3
$\mu=127, \sigma=5$	white noise	62×62	14.2	12.8	1.9	2.0	2.1	2.9	7.9	8.1	10.9	3.0	3.0	3.2
$\mu=127, \sigma=10$	white noise	62×62	17.4	17.4	15.1	3.5	3.5	3.7	15.0	15.0	15.7	2.9	2.9	2.9
$\mu=127, \sigma=15$	white noise	62×62	17.2	17.2	17.0	4.7	4.7	4.7	22.9	22.9	23.0	2.8	2.8	2.8
$\mu=127, \sigma=20$	white noise	62×62	18.8	18.8	18.7	6.6	6.6	6.6	30.5	30.5	30.6	2.8	2.8	2.8
$\mu=127, \sigma=25$	white noise	62×62	18.6	18.6	18.6	7.8	7.8	7.8	38.0	38.0	38.0	2.8	2.8	2.8
$\mu=127, \sigma=30$	white noise	62×62	18.9	18.9	18.9	9.1	9.1	9.1	45.7	45.7	45.7	2.8	2.8	2.8
$\mu=127, \sigma=35$	white noise	62×62	20.0	20.0	20.0	10.3	10.3	10.3	52.7	52.7	52.7	2.8	2.8	2.8

Table 12 - Statistical Results for Bright CP Line-Like Masks
at Pixels where Nonlinear Detector Does Not Respond

Name	Type	Size	% of Masks vs. Magnitude			Mean Robustness vs. Magnitude			Mean Contrast vs. Magnitude			Mean Fisher-distance vs. Magnitude		
			1	5	10	1	5	10	1	5	10	1	5	10
window_4	suburban	20×20	4.3	5.2	6.2	4.9	4.2	3.8	23.4	19.9	18.3	2.8	2.7	2.7
window_8	suburban	48×64	2.8	3.4	5.0	4.4	3.8	3.2	18.4	15.7	13.3	3.0	3.0	3.0
window_7	suburban	64×64	3.6	4.3	5.3	3.9	3.5	3.2	18.0	15.7	14.5	2.9	2.9	3.0
upper-left	suburban	64×64	3.4	4.9	6.7	4.9	3.8	3.4	21.7	16.2	14.2	3.0	3.1	3.1
upper-right	suburban	64×64	3.9	4.7	7.0	2.9	2.6	2.4	16.9	14.8	12.6	2.6	2.7	2.8
lower-left	suburban	64×64	3.9	5.4	6.8	4.2	3.3	3.1	18.4	14.3	13.3	2.8	2.9	2.9
lower-right	suburban	64×64	3.0	4.1	5.7	3.5	2.9	2.6	17.9	14.4	12.4	2.8	2.8	2.8
window_1	homogeneous	16×16	1.5	10.7	10.7	1.0	1.0	1.0	2.6	2.5	2.5	3.7	3.7	3.7
window_2	homogeneous	16×24	2.6	6.2	8.8	1.1	1.2	1.3	4.4	3.9	4.5	2.6	3.0	2.9
window_3	homogeneous	16×24	2.6	6.5	9.1	2.4	1.6	1.7	7.8	5.6	6.1	3.0	3.2	3.0
window_5	homogeneous	64×64	3.5	8.8	11.0	1.2	1.2	1.4	4.1	3.7	4.4	∞	∞	∞
window_6	homogeneous	64×64	3.4	9.6	11.2	1.2	1.2	1.3	3.6	3.3	3.8	3.3	∞	∞
$\mu=127, \sigma=5$	white noise	62×62	3.2	4.6	15.5	1.8	1.6	1.8	7.2	6.7	7.4	2.9	2.9	2.9
$\mu=127, \sigma=10$	white noise	62×62	3.5	3.6	5.9	3.3	3.3	2.8	14.6	14.5	12.9	3.0	3.0	2.8
$\mu=127, \sigma=15$	white noise	62×62	3.2	3.2	3.4	4.5	4.5	4.3	22.0	22.0	21.5	2.7	2.7	2.7
$\mu=127, \sigma=20$	white noise	62×62	3.4	3.4	3.5	5.0	5.0	5.0	27.8	27.8	27.6	2.7	2.7	2.7
$\mu=127, \sigma=25$	white noise	62×62	3.4	3.4	3.4	7.0	7.0	7.0	35.6	35.6	35.6	2.8	2.8	2.8
$\mu=127, \sigma=30$	white noise	62×62	3.3	3.3	3.3	8.1	8.1	8.1	40.0	40.0	40.0	2.8	2.8	2.8
$\mu=127, \sigma=35$	white noise	62×62	3.4	3.4	3.4	7.8	7.8	7.8	47.3	47.3	47.3	2.6	2.6	2.6

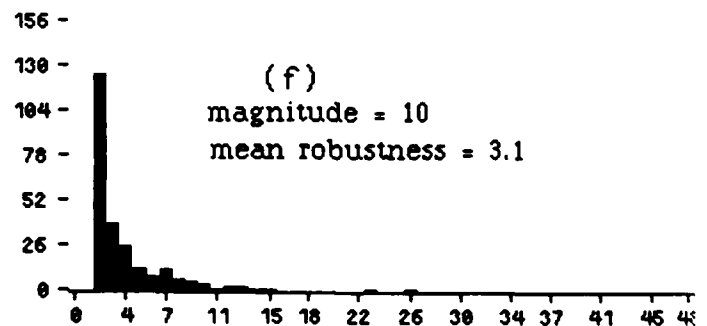
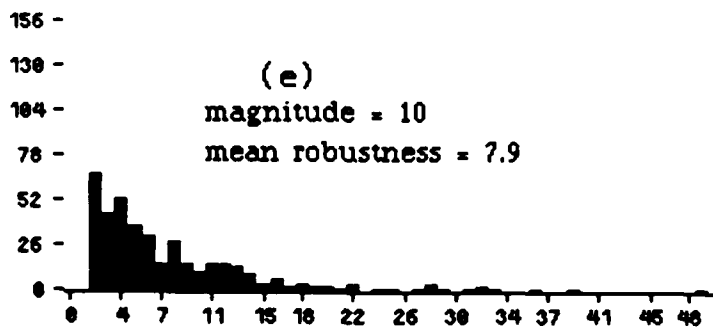
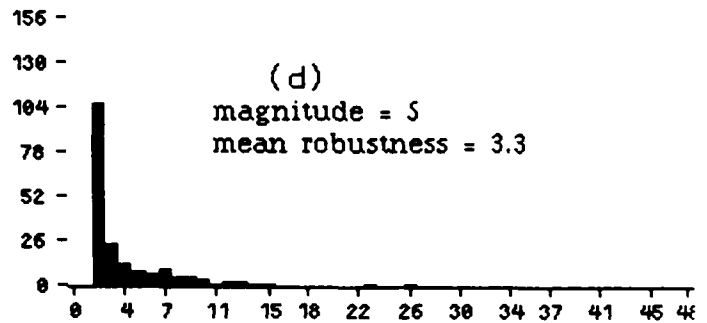
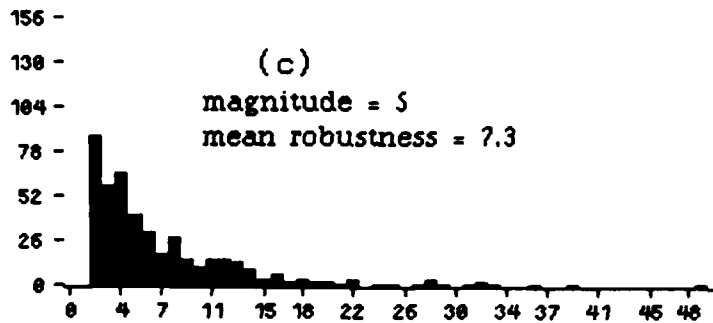
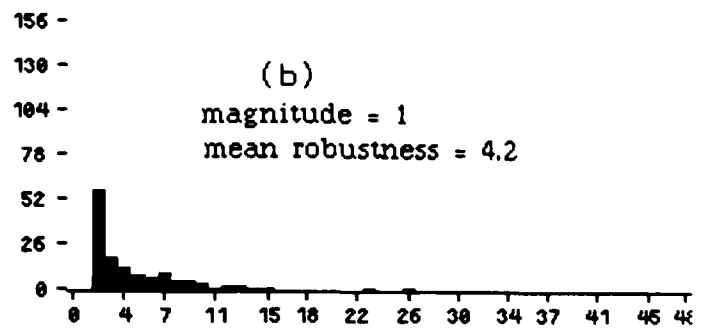
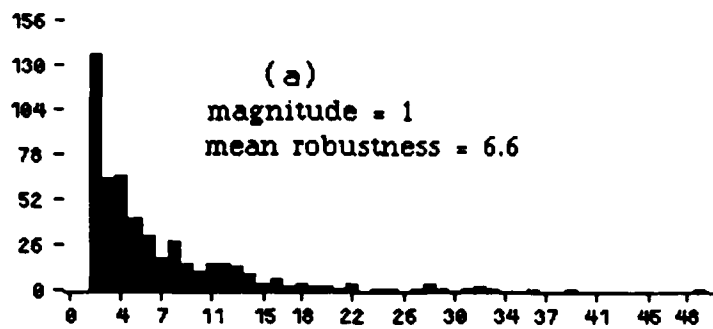


Figure 15a. Robustness Histograms for Bright CP Line-Like Masks in Lower-left Quadrant.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

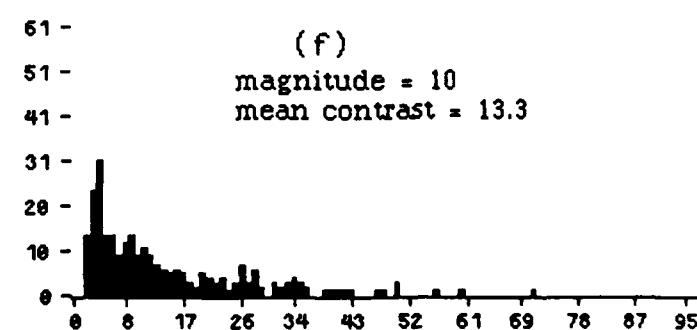
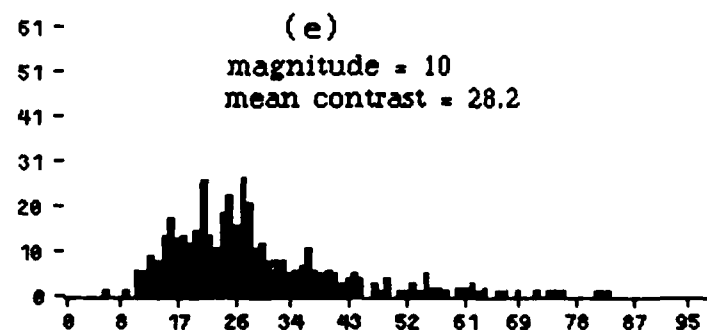
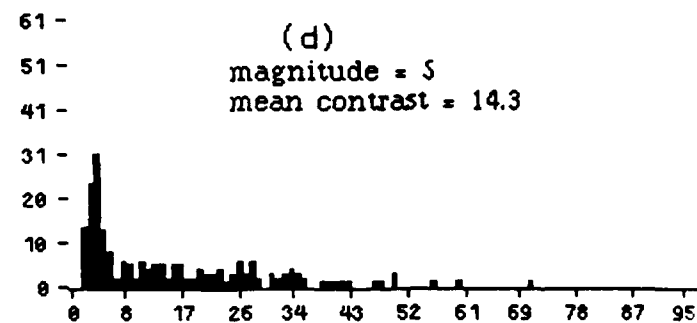
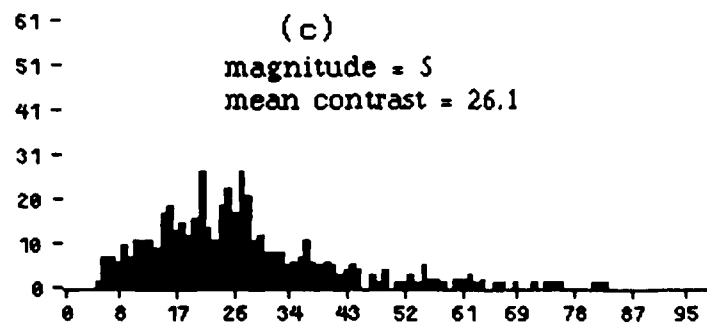
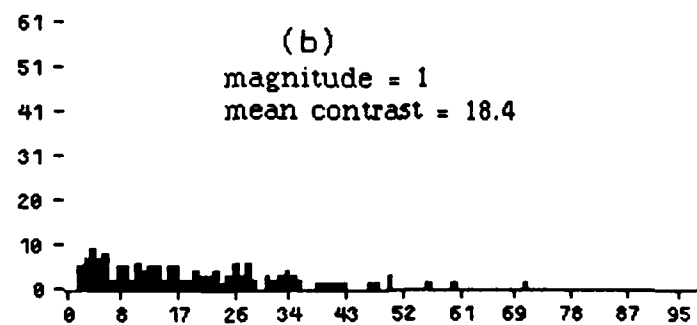
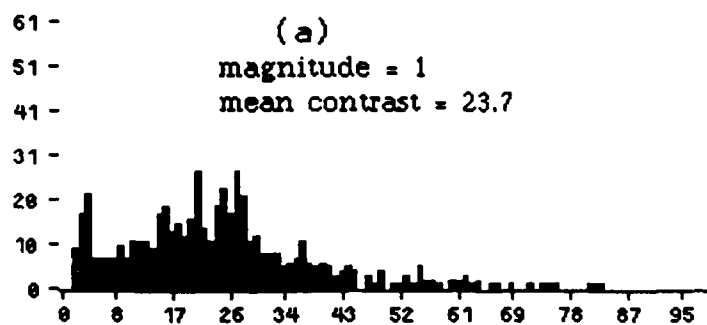


Figure 15b. Contrast Histograms for Bright CP Line-Like Masks in Lower-left Quadrant.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

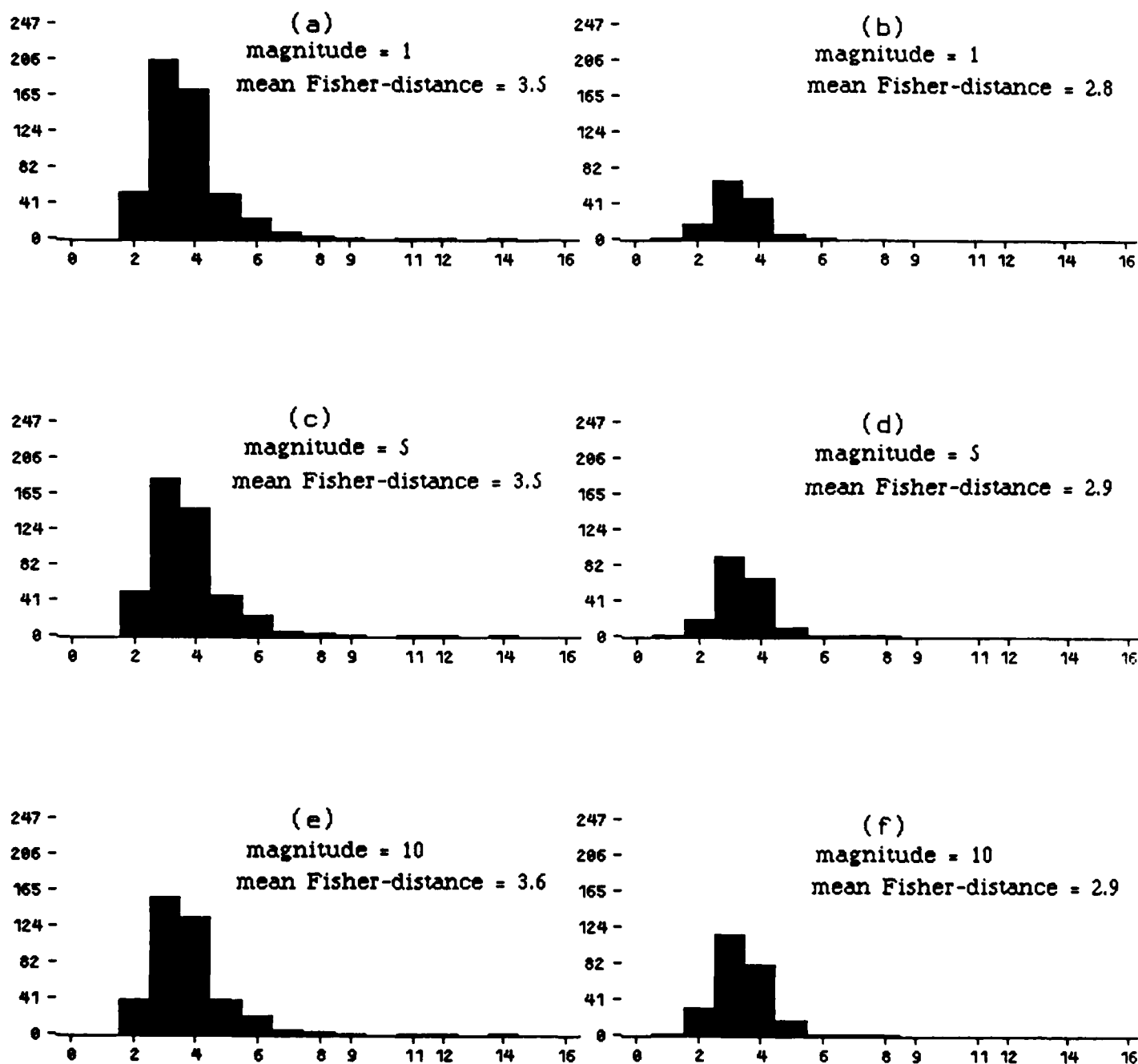


Figure 15c. Fisher-distance Histograms for Bright CP Line-Like Masks in Lower-left Quadrant.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

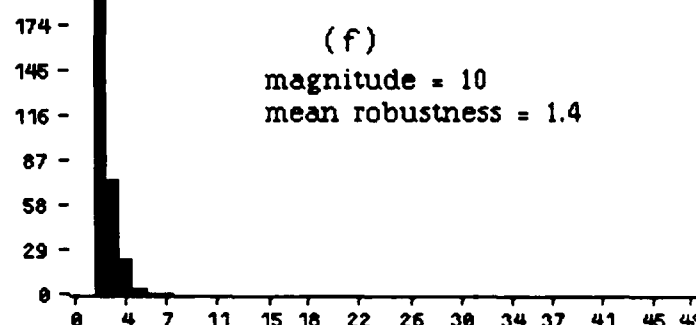
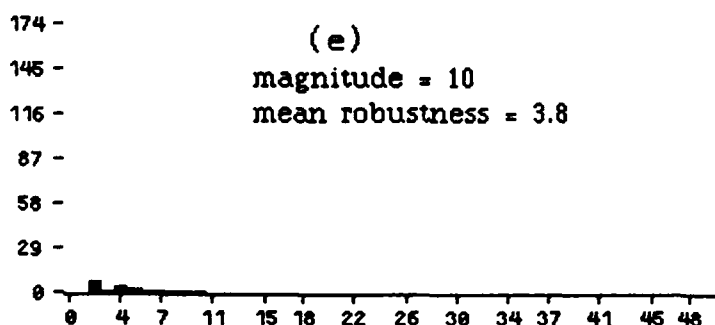
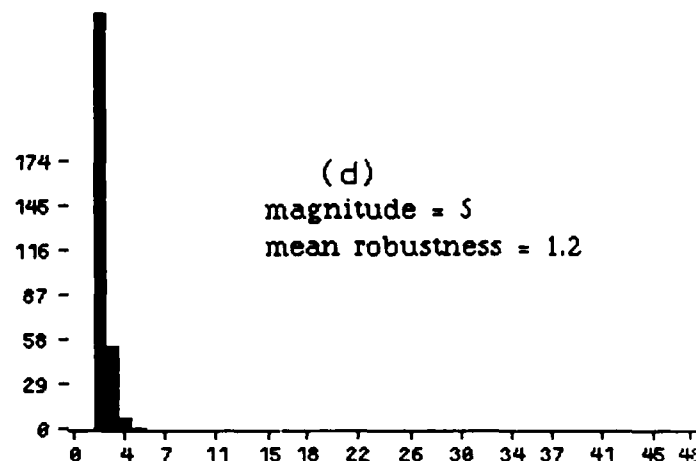
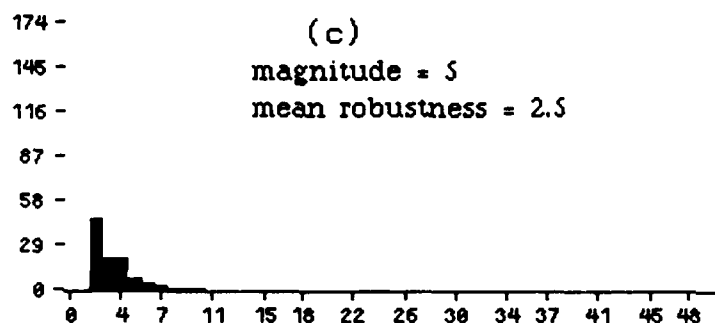
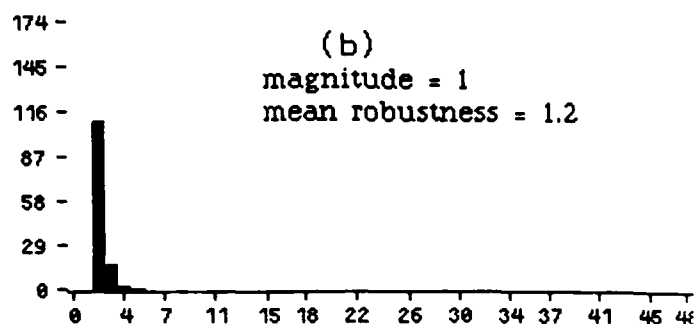
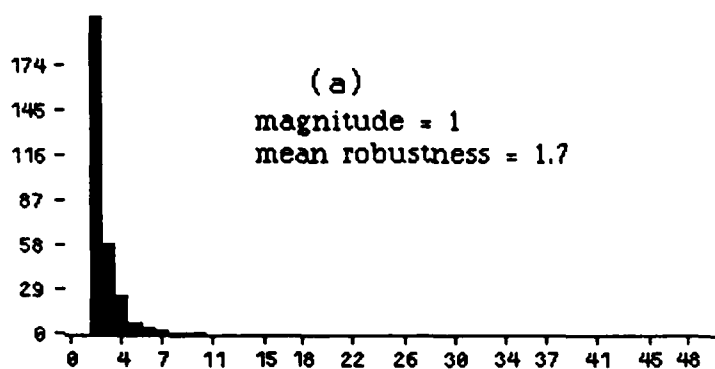


Figure 16a. Robustness Histograms for Bright CP Line-Like Masks in window_5.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

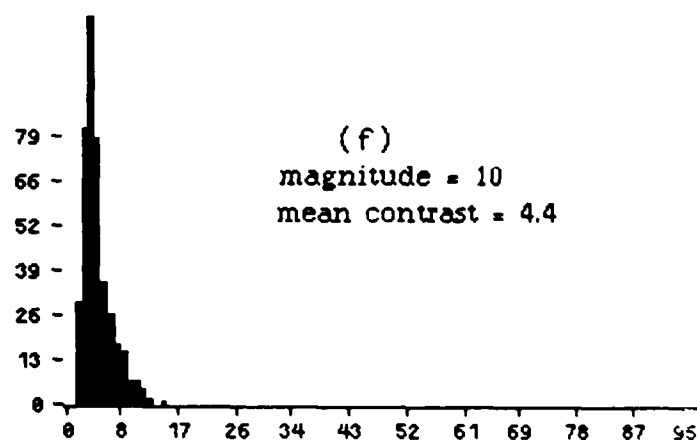
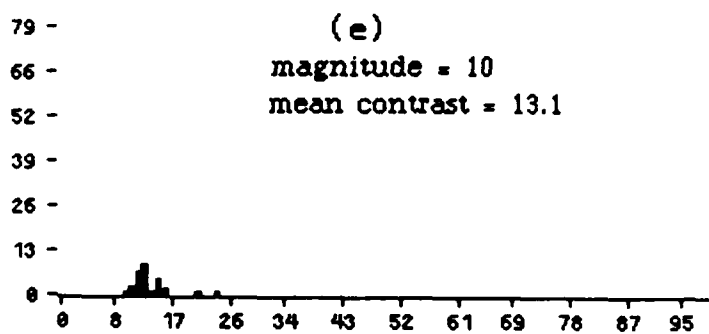
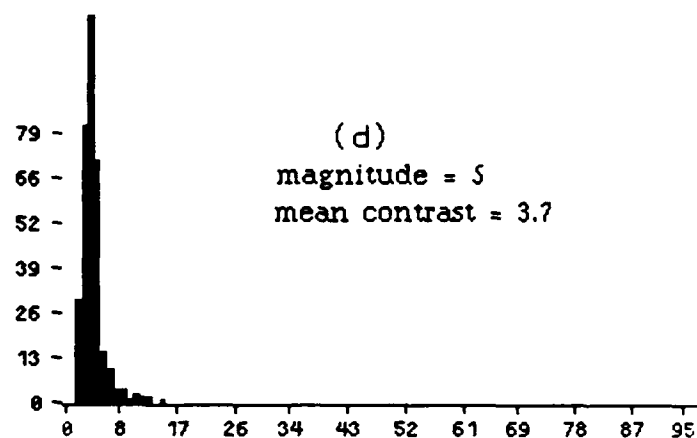
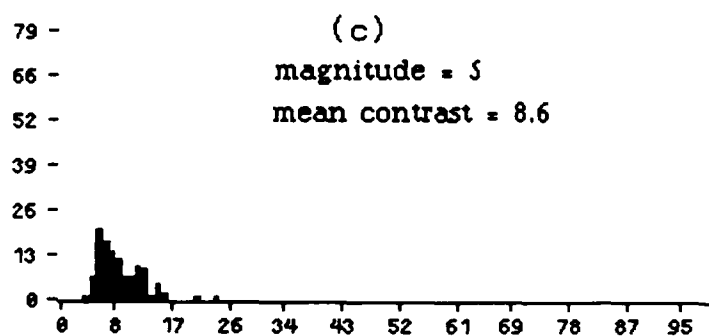
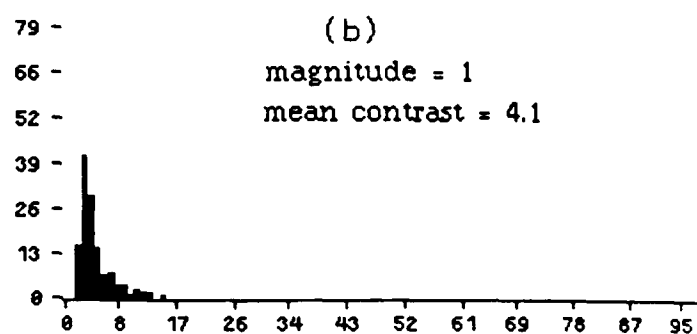
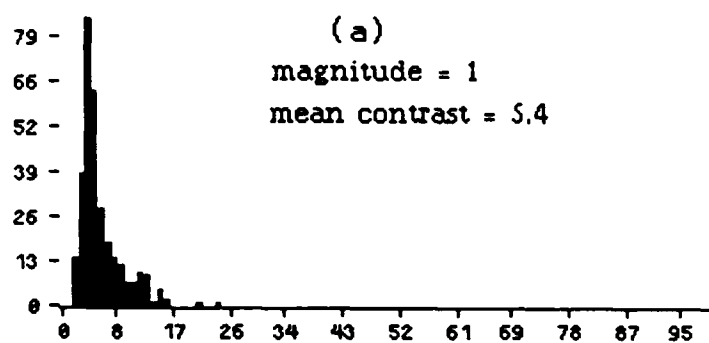


Figure 16b. Contrast Histograms for Bright CP Line-Like Masks in window_5.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

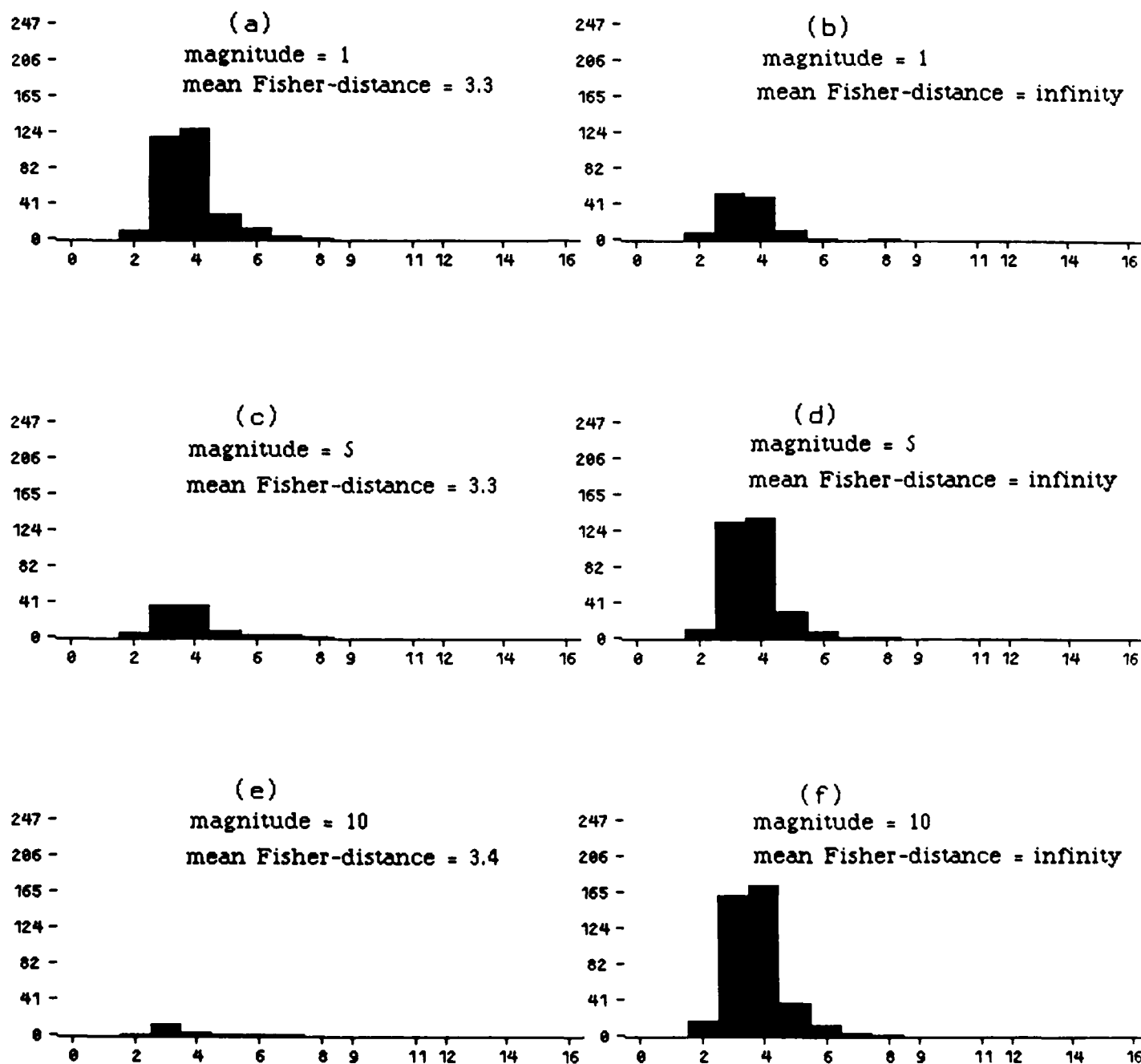


Figure 16c. Fisher-distance Histograms for Bright CP Line-Like Masks in window_5.

- (a), (c), (e): at pixels where nonlinear detector responds.
 (b), (d), (f): at pixels where nonlinear detector does not respond.

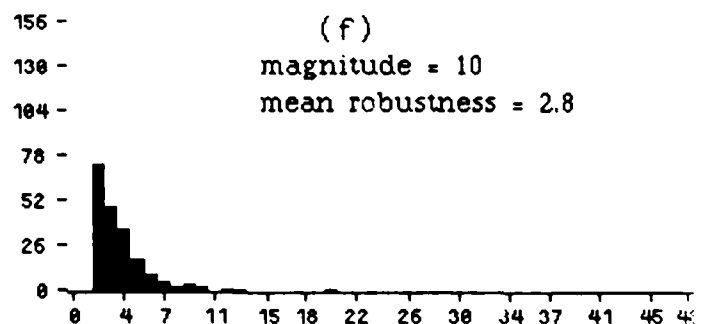
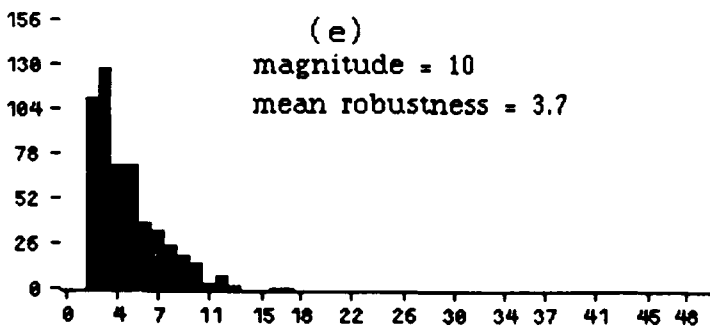
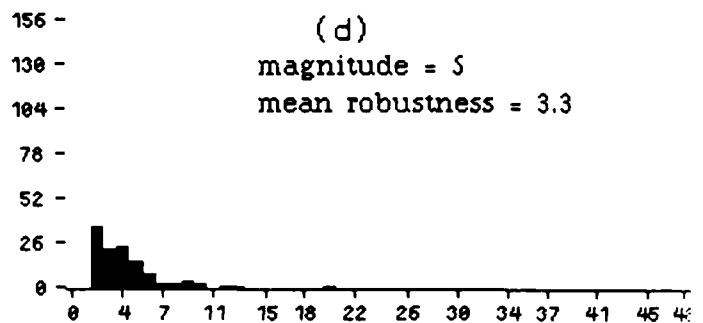
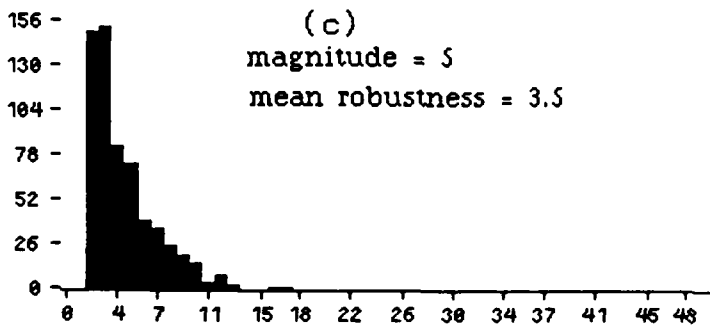
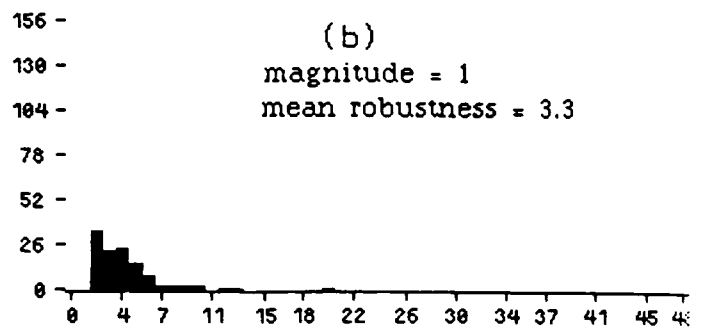
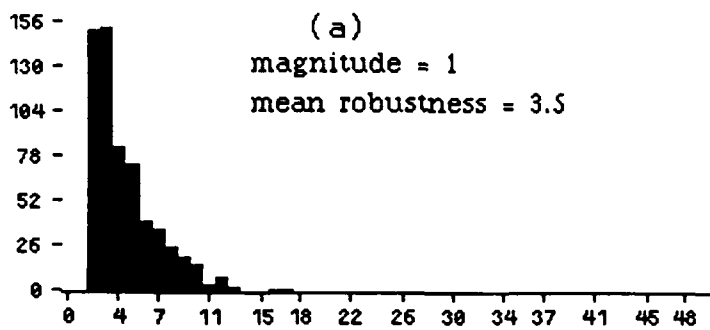


Figure 17a. Robustness Histograms for Bright CP Line-Like Masks in "White Noise" ($\sigma=10$) Image.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

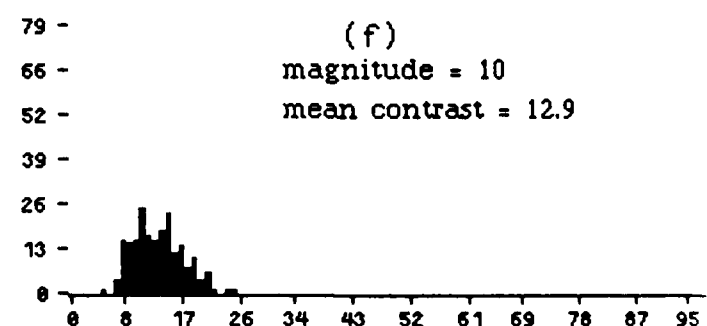
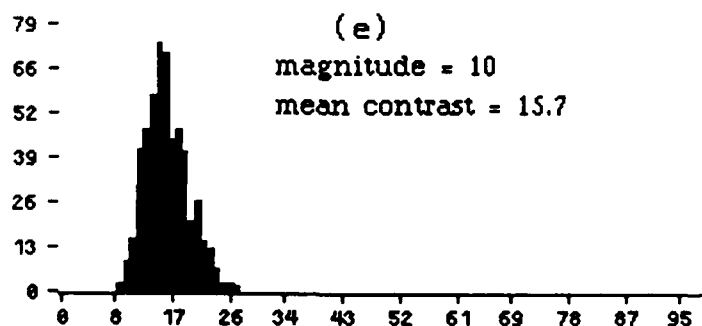
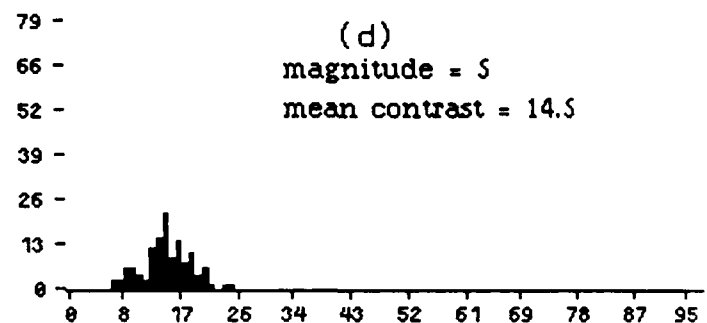
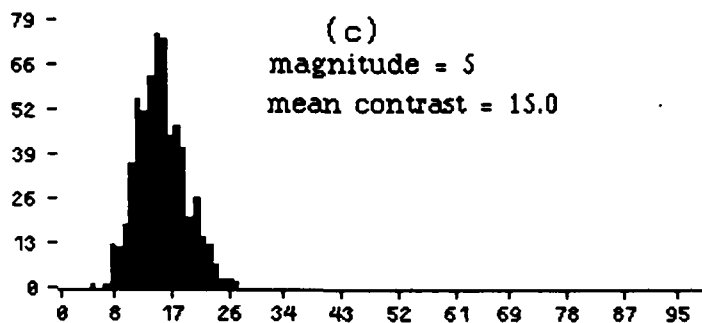
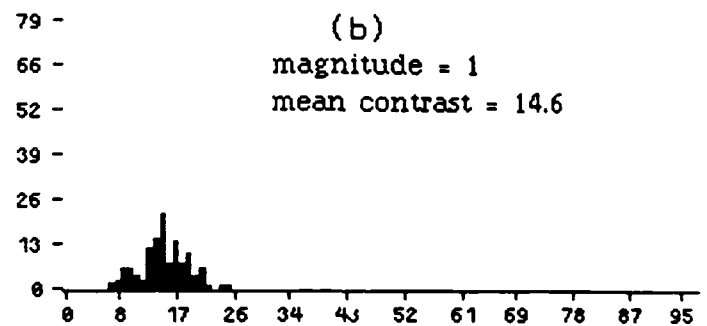
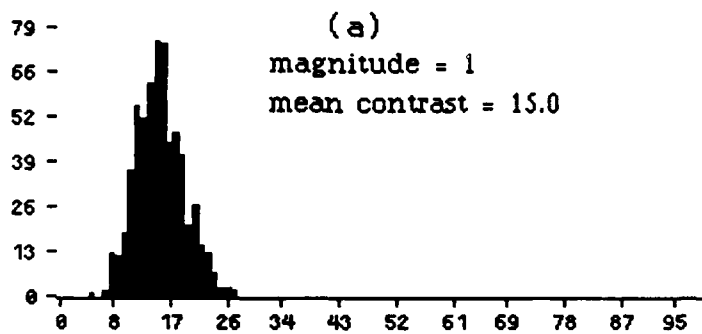


Figure 17b. Contrast Histograms for Bright CP Line-Like Masks in "White Noise" ($\sigma=10$) Image.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

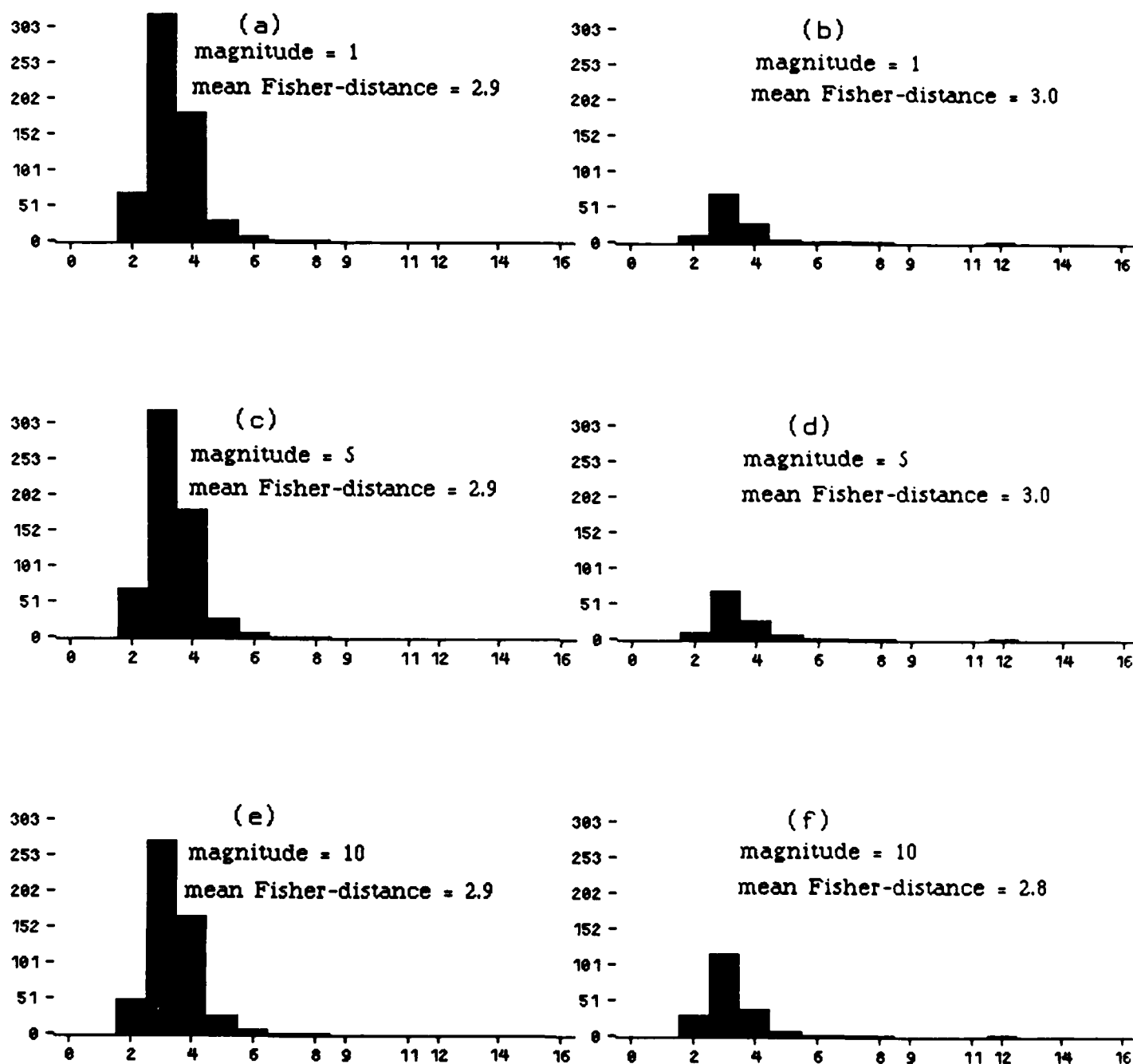


Figure 17c. Fisher-distance Histograms for Bright CP Line-Like Masks in "White Noise" ($\sigma=10$) Image.

(a), (c), (e): at pixels where nonlinear detector responds.

(b), (d), (f): at pixels where nonlinear detector does not respond.

4.3. Summary of Experimental Results

- (1) The statistics tend to be characteristic of images of a given type. However, the statistical properties of the white noise images (of appropriate variance) overlap those of the suburban images.
- (2) The frequencies of occurrence of classes of matched masks generally do not depend strongly on the type of the image. (Exceptions are the End and Corner mask classes.)
- (3) Images of suburban type can in principle be distinguished from images of homogeneous type by the values of parameters such as the mean and median robustness or contrast of the matched masks.
- (4) Images of suburban type may sometimes be distinguished from white noise images by comparing the statistics of bright CP and dark CP matched masks.

5. Probabilistic Analysis

5.1. Motivation and Problem Definition

The objective of the analysis in this section is to compute the probabilities of certain events (e.g., frequency of occurrence of a given mask, expected robustness of a class of masks, etc.) for a "white noise" image having normally distributed gray levels. These probabilities can be used to identify a real image "window" as "interesting" in the sense of having a high frequency of occurrence of line-like features.

Let glv , the image gray level, be a random variable having a Gaussian distribution with mean μ and standard deviation σ . Assuming independent pixels, we want to evaluate the following:

- Probability for a 3×3 neighborhood to satisfy a given mask or class of masks.
- Expected robustness for a given mask/class of masks.

For a continuous random variable glv , the normal density function is given by

$$f(glv) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(glv-\mu)^2/2\sigma^2}$$

Since glv takes on integer values only, it is more appropriate to treat it as a random variable of *discrete* type [5]. Assuming a gray scale of 0 to 255, the density function of glv is thus

$$f(glv) = \frac{1}{\sigma\sqrt{2\pi}} \sum_{k=0}^{255} e^{-(k-\mu)^2/2\sigma^2} \delta(glv-k)$$

where δ denotes the Dirac delta or impulse function.

5.2. Theoretical Derivation

In deriving the probability of a 3×3 neighborhood satisfying a given mask, we first observe, based on the assumption that the pixels are statistically independent, that this probability is the same for any d dark pixel mask. Thus we need only derive the probability of satisfying a $\{d, 9-d\}$ mask, without reference to its specific geometric pattern.

Let $F_{glv}(t) = Pr\{glv \leq t\}$ (i.e., the probability of a pixel gray level being less than or equal to t) denote the distribution function of the random variable glv .

Obviously, $Pr\{glv \leq t\} = \sum_{k=0}^t f(glv=k)$, and $Pr\{glv \leq 255\} = 1$.

Let t, r denote the threshold and robustness of a given $\{d, 9-d\}$ mask matched by some 3×3 neighborhood, and let $\langle t, r, d \rangle$ denote this statistical event. The probability of this event is $Pr^d\{glv \leq t\}Pr^{9-d}\{glv \geq t+r\}$, provided there exists at least one "dark" pixel with $glv=t$ and at least one "bright" pixel with $glv=t+r$. Thus

$$Pr\{\langle t, r, d \rangle\} = \left[Pr^d\{glv \leq t\} - Pr^d\{glv < t\} \right] \left[Pr^{9-d}\{glv \geq t+r\} - Pr^{9-d}\{glv > t+r\} \right]$$

or, in terms of the density and distribution functions:

$$Pr\{<t, r, d>\} =$$

$$\left\{ F_{glv}^d(t) - \left[F_{glv}(t) - f(glv=t) \right]^d \right\} \left\{ \left[1 - F_{glv}(t+r) + f(glv=t+r) \right]^{9-d} - \left[1 - F_{glv}(t+r) \right]^{9-d} \right\}$$

Summation of this probability over all possible values for t and r (this is legitimate, since it involves addition of probabilities of disjoint events) yields

$$Pr\{<d>\} = \sum_{r>0}^{255} \sum_{t=0}^{255-r} Pr\{<t, r, d>\}$$

where $<d>$ stands for the event that a 3×3 neighborhood matches a given $\{d, 9-d\}$ mask.

It should be noted that since we require strictly positive robustness, we have $\binom{9}{d} Pr\{<d>\} < 1$ for $d = 1, 2, \dots, 8$. In other words, the probability of some 3×3 neighborhood matching *any* d dark pixel mask is strictly less than 1.

Also, since matching different masks simultaneously in a given 3×3 neighborhood still involves disjoint events (i.e., $<d_1> \cap <d_2> = \emptyset$, provided that $mask_number\{d_1, 9-d_1\} \neq mask_number\{d_2, 9-d_2\}$ in case $d_1=d_2$), the probability of a 3×3 neighborhood satisfying a given *class* of masks is

$$Pr\{<C_j>\} = \sum_{i=1}^8 n_i(C_j, d_i) Pr\{<d_i>\}$$

where $<C_j>$ denotes the event that the neighborhood matches a mask which is a member of class C_j , and $n_i(C_j, d_i)$ is the number of $\{d_i, 9-d_i\}$ mask configurations in class C_j (See Table 1.) For example, the probability of a 3×3 neighborhood satisfying an edge type mask is

$$Pr\{\langle Edge \rangle\} = 8 Pr\{\langle 5 \rangle\} + 8 Pr\{\langle 6 \rangle\}$$

Similarly, the probabilities for the rest of the classes can be evaluated.

In evaluating the expected robustness for a given mask, a proper normalization factor should be introduced. This implies expressing the required expected value in the following manner:

$$E(r|\langle d \rangle) = \frac{\sum_{r>0}^{255} r Pr\{\langle r, d \rangle\}}{Pr\{\langle d \rangle\}}$$

where $Pr\{\langle r, d \rangle\} = \sum_{t=0}^{255-r} Pr\{\langle t, r, d \rangle\}$ denotes the probability that the neighborhood matches a given d dark pixel mask having robustness r .

To conclude our analysis, we now extend the above expected value to a given class of masks. The result is evaluated by the following weighted sum:

$$E(r|\langle C_j \rangle) = \frac{\sum_{i=1}^8 n_i(C_j, d_i) Pr\{\langle d_i \rangle\} E(r|\langle d_i \rangle)}{Pr\{\langle C_j \rangle\}}$$

5.3. Numerical Results

The numerical values shown in the following tables were obtained for $\mu=127.5$ and $\sigma=5, 10, 15, 20, 25, 30, 35$. (Since the random variable is of discrete type, higher values for σ are impractical.) Note that $Pr\{\langle C_{j_1} \rangle\} = Pr\{\langle C_{j_2} \rangle\}$ and $E(r|\langle C_{j_1} \rangle) = E(r|\langle C_{j_2} \rangle)$ for classes of masks consisting of *single* (identical or complementary) partition types. Thus, the results for these cases are given for a combined class entry form.

Table 13 - Probability of Matching a Given $\{d, 9-d\}$ Mask vs. σ							
d	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$
1	.095724	.103145	.105738	.107058	.107857	.108374	.108526
2	.021880	.024633	.025635	.026153	.026470	.026683	.026837
3	.008889	.010269	.010783	.011052	.011217	.011328	.011409
4	.005778	.006756	.007125	.007318	.007437	.007518	.007576
5	.005778	.006756	.007125	.007318	.007437	.007518	.007576
6	.008889	.010269	.010783	.011052	.011217	.011328	.011408
7	.021880	.024633	.025635	.026153	.026470	.026683	.026836
8	.095724	.103145	.105738	.107058	.107857	.108393	.108775

Table 14 - Probability of Matching a Given Class of Masks vs. σ							
Class	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$
Edge	.117337	.136195	.143266	.146962	.149233	.150769	.151877
Line	.106668	.123222	.129399	.132622	.134600	.135938	.136903
Wide-Line	.023113	.027024	.028500	.029274	.029750	.030072	.030305
Right-Ang. Acute-Ang.	.071112	.082148	.086266	.088415	.089733	.090625	.091269
End-Point	.175040	.197064	.205082	.209227	.211757	.213464	.214694
Corner	.581048	.636667	.656534	.666731	.672935	.677106	.680093
L-Line Y-Junction	.046225	.054047	.057000	.058548	.059500	.060144	.060610
T-Junction	.156858	.179603	.188041	.192435	.195128	.196948	.198259
X-Junction	.011556	.013512	.014250	.014637	.014875	.015036	.015152

Table 15 - Expected Robustness of a Given $\{d, 9-d\}$ Mask vs. σ							
d	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$
1	3.20248	5.95157	8.71042	11.47166	14.23372	16.98960	19.68012
2	2.28344	4.06157	5.85552	7.65334	9.45269	11.25332	13.06100
3	1.98847	3.44697	4.92478	6.40733	7.89176	9.37728	10.86542
4	1.88220	3.22365	4.58604	5.95359	7.32318	8.69383	10.06560
5	1.88220	3.22365	4.58604	5.95359	7.32318	8.69375	10.06437
6	1.98848	3.44697	4.92478	6.40733	7.89176	9.37697	10.86064
7	2.28345	4.06158	5.85553	7.65335	9.45270	11.25231	13.04562
8	3.20247	5.95157	8.71042	11.47166	14.23372	16.98970	19.68163

Table 16 - Expected Robustness of a Given Class of Masks vs. σ							
Class	$\sigma=5$	$\sigma=10$	$\sigma=15$	$\sigma=20$	$\sigma=25$	$\sigma=30$	$\sigma=35$
Edge	1.94661	3.35835	4.79001	6.22657	7.66507	9.10422	10.54287
Line	1.88220	3.22365	4.58604	5.95359	7.32318	8.69375	10.06437
Right-Ang.							
Acute-Ang.							
Wide-Line							
L-Line	2.28344	4.06157	5.85552	7.65334	9.45269	11.25332	13.06100
X-Junction							
Y-Junction							
End-Point	2.87310	5.25078	7.63959	10.03116	12.42373	14.81250	17.15832
Corner							
T-Junction							

Theoretically, the probability and expected robustness results for a given $\{d, 9-d\}$ mask should be identical to those for a given $\{9-d, d\}$ mask. (Practically, the results are identical within minor inaccuracies for the higher σ 's examined.)

Also, the agreement between the theoretical results and those obtained in Section 4 for "white noise" images should be noted. For example, for $\sigma=5$, $Pr\{\langle Corner \rangle\} = 0.581048$, whereas the relative average frequency of occurrence of Corner type masks, in the corresponding white-noise image, is approximately 59% . Similarly, for $\sigma=20$, $E(r|\langle Line \rangle) = 6.41$, whereas the average mean robustness computed for Line type masks in the corresponding white-noise image is approximately 6.35 .

Finally, the σ dependence of $Pr\{\langle d \rangle\}$ (i.e., $\binom{9}{d} Pr\{\langle d \rangle\} \rightarrow 1$ as σ approaches its maximum possible value) is qualitatively explained by the following argument. The higher σ , the less probable it is that two pixels have the same gray level value. Thus, the frequency of occurrence of masks having zero robustness decreases, and consequently, the probability of a 3×3 neighborhood matching any non-zero robustness mask approaches 1.

6. Concluding Remarks

In this paper, we have introduced a set of 3×3 binary masks that can be used to extract linear features from an image. By matching the masks with the image, we assign labels to each pixel, each label corresponding to a particular mask. We also compute confidence measures associated with each label, based on the homogeneity of the foreground and background in each mask, and the difference between them. Several statistical studies, attempting to distinguish suburban images from homogeneous and white noise images, were carried out.

It was shown that suppression of matched masks by thresholding of the confidence measures yields unsatisfactory results. It is preferable to record all the information obtained from the masks, in order to make use of it at the later stages of the linear feature detection process.

7. Acknowledgements

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Appendix A - 512 3 × 3 Binary Masks

Appendix B - Statistics of Additional Images.

Table B-1a - Statistics for Dark Center Pixel Masks in Lower-right Quadrant of Greenbelt I

Class	# of Masks	Robustness				Contrast				Fisher distance			
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	2104	127.0	8.2	174.8	4.0	163.3	25.8	502.4	20.5	24.7	3.7	4.4	3.2
Line	318	49.0	5.6	62.0	2.0	112.3	21.0	403.1	15.6	16.7	2.8	3.4	2.4
Wide-Line	79	10.0	2.3	3.4	1.0	59.4	16.4	141.2	12.6	5.4	2.6	0.8	2.5
Right-Angle	134	25.0	3.2	14.7	2.0	87.2	17.4	211.3	13.2	6.1	2.3	0.8	2.2
Acute-Angle	300	18.0	3.2	10.5	2.0	86.2	17.7	175.8	14.8	8.0	2.4	1.0	2.2
End-Point	489	27.0	3.4	12.9	2.0	97.4	18.4	260.2	14.4	8.5	2.2	0.8	2.0
Corner	3782	103.0	8.2	117.7	4.0	148.5	24.9	450.7	19.5	47.5	3.3	4.8	2.8
L-Line	187	23.0	3.9	17.1	2.0	120.6	22.1	423.3	16.8	6.1	2.8	0.8	2.7
T-Junction	504	74.0	4.1	42.6	2.0	138.3	20.7	345.8	16.5	8.1	2.9	1.1	2.8
X-Junction	12	3.0	1.8	0.5	2.0	30.2	14.7	91.5	13.2	3.2	2.4	0.2	2.4
Y-Junction	47	8.0	2.6	4.7	2.0	38.6	15.5	86.9	15.2	3.8	2.5	0.4	2.5

Table B-1b - Statistics for Bright Center Pixel Masks in Lower-right Quadrant of Greenbelt I

Class	# of Masks	Robustness				Contrast				Fisher distance			
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	2126	148.0	8.1	189.3	4.0	168.2	26.1	508.5	21.1	38.6	3.3	5.3	2.9
Line	320	40.0	6.4	54.9	3.0	109.2	23.5	320.6	19.8	18.5	3.3	3.0	2.8
Wide-Line	89	23.0	3.7	19.6	2.0	62.3	19.6	195.2	16.3	8.0	2.8	1.1	2.6
Right-Angle	164	19.0	3.2	12.2	2.0	90.0	19.4	229.9	15.9	6.7	2.6	0.7	2.5
Acute-Angle	332	61.0	3.9	31.9	2.0	113.7	20.8	310.8	16.6	6.6	2.8	0.8	2.7
End-Point	517	36.0	5.6	35.5	3.0	99.3	22.6	311.5	18.2	16.7	2.8	1.5	2.6
Corner	3830	96.0	5.8	60.5	3.0	147.1	22.8	375.9	18.1	41.3	2.5	1.9	2.2
L-Line	209	27.0	3.4	12.1	2.0	115.2	21.7	368.1	18.6	5.9	2.9	0.6	2.8
T-Junction	490	35.0	3.5	17.3	2.0	125.9	19.7	295.9	16.0	6.8	2.5	0.8	2.4
X-Junction	10	4.0	1.8	1.4	1.0	50.8	18.2	212.0	16.7	4.1	2.6	0.6	2.3
Y-Junction	42	13.0	3.6	12.4	2.0	38.1	17.3	81.8	16.7	4.3	2.8	0.4	2.8

Table B-2a - Statistics for Dark Center Pixel Masks in 16X16 window_1 of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Var
Edge	26	2.0	1.1	0.1	3.8	2.7	0.5	4.6	3.2	0.5
Line	19	3.0	1.2	0.2	5.8	2.9	1.7	5.7	3.6	0.7
Wide-Line	3	1.0	1.0	0.0	2.3	1.9	0.1	3.2	2.9	0.2
Right-Angle	2	1.0	1.0	0.0	2.2	2.2	0.0	5.3	4.4	0.8
Acute-Angle	7	2.0	1.1	0.1	3.8	2.4	0.7	4.2	3.5	0.3
End-Point	23	2.0	1.3	0.2	6.1	3.0	1.8	5.9	3.2	1.0
Corner	91	3.0	1.2	0.2	4.4	2.6	0.4	5.2	2.8	0.6
L-Line	6	2.0	1.2	0.1	4.8	2.7	1.7	4.0	3.3	0.4
T-Junction	17	2.0	1.1	0.1	4.6	2.4	0.7	6.8	3.4	1.2
X-Junction	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y-Junction	4	1.0	1.0	0.0	3.1	2.5	0.3	6.0	4.2	1.3

Table B-2b - Statistics for Bright Center Pixel Masks in 16X16 window_1 of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Var
Edge	29	4.0	1.3	0.4	6.2	3.3	1.2	6.5	3.5	0.8
Line	11	2.0	1.1	0.1	4.0	2.5	0.7	7.1	4.0	2.7
Wide-Line	4	1.0	1.0	0.0	3.0	2.4	0.3	3.8	3.6	0.0
Right-Angle	3	1.0	1.0	0.0	2.7	2.2	0.1	2.5	2.4	0.0
Acute-Angle	13	2.0	1.1	0.1	3.2	2.3	0.3	6.1	3.1	1.1
End Point	18	2.0	1.1	0.1	4.4	2.5	0.8	6.9	3.1	2.0
Corner	109	4.0	1.4	0.4	7.0	3.2	1.6	6.6	3.1	0.9
L Line	6	1.0	1.0	0.0	4.4	2.5	0.9	3.5	3.0	0.1
T Junction	16	2.0	1.1	0.1	4.6	2.5	0.8	4.3	3.1	0.3
X Junction	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y Junction	2	1.0	1.0	0.0	1.9	1.8	0.0	6.1	5.0	1.2

Table B 3a Statistics for Dark Center Pixel Masks in 16X24 window_2 of Greenbelt1												
Class	# of Masks	Robustness			Contrast			Fisher distance			Med	Max
		Max	Mean	Var	Med	Nlax	Mean	Var	Med	Max		
Edge	170	74.0	8.7	250.1	3.0	85.0	21.1	472.5	11.1	28.5	4.8	17.1
Line	26	3.0	1.3	0.4	1.0	36.8	6.1	43.9	4.5	4.1	2.7	0.9
Wide Line	3	1.0	1.0	0.0	1.0	5.1	4.3	0.9	4.8	3.7	3.0	0.3
Right Angle	4	2.0	1.3	0.2	1.0	12.3	6.0	13.5	4.2	3.5	2.8	0.7
Acute Angle	12	2.0	1.2	0.1	1.0	9.0	5.7	5.7	4.7	3.8	2.5	0.5
End Point	32	4.0	1.3	0.4	1.0	35.8	7.4	67.6	4.6	4.5	2.5	0.8
Corner	310	46.0	4.6	39.6	2.0	75.2	16.3	396.6	7.1	13.6	3.1	1.7
T Line	21	3.0	1.5	0.3	1.0	48.8	6.8	91.1	4.4	4.1	2.9	0.4
T Junction	21	6.0	1.4	1.2	1.0	70.3	8.5	208.0	4.5	4.3	3.1	0.3
X Junction	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y Junction	2	1.0	1.0	0.0	1.0	4.1	2.9	1.4	2.9	3.5	3.2	0.1

Table B 3b Statistics for Bright Center Pixel Masks in 16X24 window_2 of Greenbelt1												
Class	# of Masks	Robustness			Contrast			Fisher distance			Med	Max
		Max	Mean	Var	Med	Nlax	Mean	Var	Med	Max		
Edge	164	68.0	4.3	56.9	2.0	75.8	17.7	274.2	10.5	20.5	3.3	4.7
Line	23	31.0	5.0	65.9	1.0	57.7	19.6	394.2	5.5	6.9	3.2	1.8
Wide Line	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Right Angle	6	2.0	1.2	0.1	1.0	5.0	3.7	1.0	3.9	4.9	3.1	0.7
Acute Angle	20	11.0	2.4	8.5	1.0	59.2	14.6	360.9	5.3	3.7	2.7	0.4
End Point	31	29.0	3.6	34.8	1.0	58.6	16.0	386.9	5.4	5.1	2.7	0.9
Corner	239	23.0	2.3	6.2	2.0	66.1	12.7	188.9	6.5	6.9	2.5	1.0
T Line	20	45.0	4.8	107.9	1.0	66.7	19.7	465.4	6.4	10.8	3.3	3.3
T Junction	31	3.0	1.5	0.3	1.0	49.7	8.9	118.2	5.3	5.0	2.9	1.1
X Junction	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y Junction	3	2.0	1.7	0.2	2.0	4.6	3.9	0.7	4.5	4.5	3.9	0.5

Table B 4a Statistics for Dark Center Pixel Masks in 16X24 window_3 of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	129	710	51	798	20	1017	200	450.4	132	110
Line	14	60	17	18	10	225	78	346	59	34
Wide Line	6	250	53	779	10	999	196	1295.4	30	38
Right Angle	6	30	18	08	15	110	76	96	84	39
Acute Angle	24	60	22	26	10	668	127	182.2	86	47
End Point	38	70	20	26	10	599	92	105.4	61	42
Corner	289	890	88	1968	40	1288	211	715.1	114	472
L Line	14	120	21	77	10	681	118	316.6	40	40
T Junction	29	80	18	22	10	489	122	130.7	65	46
X Junction	1	20	20	00	20	58	58	00	58	31
Y Junction	8	10	10	00	10	68	34	19	31	43

Table B 4b Statistics for Bright Center Pixel Masks in 16X24 window_3 of Greenbelt1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	133	310	35	208	20	924	179	358.0	119	70
Line	19	150	25	121	10	342	76	53.2	45	56
Wide Line	3	100	67	116	80	789	409	878.7	373	37
Right Angle	8	250	115	109.3	65	942	308	787.1	25.7	63
Acute Angle	29	410	58	89.2	20	1037	217	791.3	113	86
End Point	39	460	54	117.0	10	1041	166	615.5	50	75
Corner	266	190	29	63	20	800	140	217.4	81	83
L Line	16	190	28	189	10	792	144	365.5	71	40
T Junction	28	90	21	37	10	701	127	227.9	77	50
X Junction	0	00	00	00	00	00	00	00	00	00
Y Junction	5	10	10	00	10	43	32	07	32	38

Table B-5a - Statistics for Dark Center Pixel Masks in 20X20 window_4 of Greenbelt 1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
End	121	50.0	7.5	72.6	4.0	77.0	32.6	337.0	31.9	12.1
Line	40	29.0	8.0	54.0	4.0	51.0	27.6	122.2	29.6	5.1
Wide-Line	6	5.0	3.0	1.3	3.0	41.8	26.8	143.2	26.1	4.1
Right-Angle	10	14.0	3.7	14.6	2.0	52.3	25.2	187.2	22.8	3.3
Acute-Angle	32	12.0	4.0	12.3	2.0	54.8	26.6	167.0	27.3	3.4
End-Point	52	15.0	3.7	9.7	3.0	48.4	23.9	131.7	24.6	3.3
Corner	297	66.0	15.6	253.4	10.0	98.3	38.6	516.5	36.9	21.3
L-Line	32	38.0	6.2	49.8	4.0	53.6	33.9	157.0	34.1	9.4
T-Junction	40	45.0	7.8	98.2	3.5	71.4	29.7	322.1	29.9	8.0
X-Junction	1	1.0	1.0	0.0	1.0	5.6	5.6	0.0	5.6	2.4
Y-Junction	4	14.0	6.0	23.5	4.5	35.7	28.3	55.7	30.6	4.5

Table B-5b - Statistics for Bright Center Pixel Masks in 20X20 window_4 of Greenbelt 1

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
End	144	38.0	6.3	43.6	4.0	67.6	30.1	263.5	28.9	13.2
Line	43	37.0	9.5	95.0	5.0	68.0	34.9	384.6	35.2	9.3
Wide-Line	3	3.0	2.3	0.9	3.0	34.2	29.1	36.8	32.6	2.4
Right-Angle	17	20.0	6.5	31.8	6.0	57.3	36.4	183.1	40.5	4.2
Acute-Angle	33	20.0	4.8	20.0	4.0	62.7	32.3	244.3	32.0	6.1
End-Point	65	56.0	13.2	187.2	9.0	78.7	42.5	360.8	46.8	13.1
Corner	304	30.0	5.5	29.7	4.0	61.6	27.0	192.2	27.4	5.0
L-Line	39	29.0	6.3	41.7	4.0	63.3	27.8	231.1	26.8	7.0
T-Junction	49	13.0	3.6	8.0	3.0	57.9	27.7	161.9	26.5	4.8
X-Junction	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Y-Junction	3	12.0	4.7	26.9	1.0	39.2	15.3	284.0	3.6	3.8

Table B-6a - Statistics for Dark Center Pixel Masks in 48X64 window_8 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1776	69.0	8.5	98.8	5.0	125.5	28.5	367.2	25.5	32.4
Line	224	29.0	4.2	24.5	2.0	90.5	19.9	254.4	16.7	11.7
Wide-Line	24	14.0	3.3	12.2	2.0	43.3	20.1	133.5	19.1	5.5
Right-Angle	91	18.0	2.6	10.3	1.0	60.7	15.0	143.7	12.7	6.1
Acute-Angle	240	29.0	3.5	18.1	2.0	75.5	18.5	164.5	15.8	6.2
End-Point	343	38.0	3.6	19.7	2.0	81.1	16.7	175.3	13.4	6.3
Corner	3083	141.0	8.7	124.1	5.0	160.3	27.3	415.0	23.8	39.2
L-Line	182	30.0	4.1	24.5	2.0	91.1	19.9	230.5	17.3	7.1
T-Junction	391	44.0	3.8	22.4	2.0	94.2	19.8	235.9	16.7	16.9
X-Junction	1	2.0	2.0	0.0	2.0	13.7	13.7	0.0	13.7	3.3
Y-Junction	35	10.0	2.4	4.3	2.0	53.6	13.3	119.8	11.6	3.8

Table B-6b - Statistics for Bright Center Pixel Masks in 48X64 window_8 of Greenbelt

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	1607	80.0	7.6	80.6	4.0	135.3	27.9	322.5	26.2	23.0
Line	257	69.0	6.4	57.9	4.0	128.3	24.2	272.5	21.8	10.4
Wide-Line	40	30.0	3.9	26.4	2.0	50.7	23.1	186.2	21.7	5.9
Right-Angle	105	17.0	3.1	8.5	2.0	92.3	17.8	194.0	16.0	10.2
Acute-Angle	267	50.0	4.8	35.0	3.0	123.2	24.2	302.6	21.3	9.9
End-Point	412	111.0	6.4	96.6	3.0	157.6	24.3	350.3	21.8	12.1
Corner	2830	67.0	5.4	47.4	3.0	127.3	23.9	280.2	21.6	22.0
L-Line	232	32.0	5.6	32.7	4.0	110.5	25.0	210.6	23.7	10.8
T-Junction	377	40.0	3.8	20.7	2.0	88.9	21.6	214.5	18.9	7.9
X-Junction	5	4.0	2.0	1.2	2.0	44.4	17.0	277.5	5.0	3.4
Y-Junction	33	7.0	2.6	3.0	2.0	41.7	12.9	104.2	10.3	9.9

Table B-7a - Statistics for Dark Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=5$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	473	8.0	2.0	1.7	2.0	15.5	7.7	3.9	7.5	6.9
Line	397	6.0	2.0	1.4	2.0	14.7	7.9	4.2	7.8	10.2
Wide-Line	68	6.0	1.9	1.4	2.0	11.8	7.8	3.6	7.8	5.9
Right-Angle	251	8.0	2.0	1.7	2.0	15.7	7.8	4.0	7.5	7.5
Acute-Angle	288	7.0	2.0	1.6	2.0	15.5	7.8	4.4	7.3	7.5
End-Point	674	11.0	2.4	2.4	2.0	16.7	8.1	5.1	7.9	6.9
Corner	2282	14.0	2.8	4.3	2.0	19.8	8.4	6.9	8.0	11.0
L-Line	197	6.0	1.9	1.5	2.0	15.8	7.9	3.9	7.8	5.4
T-Junction	588	9.0	2.2	2.1	2.0	15.7	7.9	4.2	7.8	8.5
X-Junction	39	6.0	1.9	1.4	1.0	11.7	7.5	3.5	7.4	6.5
Y-Junction	179	8.0	1.9	1.7	2.0	15.3	7.7	4.2	7.8	7.4

Table B-7b - Statistics for Bright Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=5$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	482	8.0	1.9	1.3	2.0	14.8	7.9	4.1	7.7	6.6
Line	403	8.0	2.0	1.6	2.0	15.8	7.9	4.3	7.7	6.2
Wide Line	78	6.0	1.7	1.0	1.0	12.9	7.5	4.1	7.2	5.1
Right Angle	276	7.0	2.0	1.6	2.0	14.8	7.9	4.3	7.7	7.3
Acute Angle	280	8.0	2.0	1.7	2.0	15.2	7.8	4.5	7.5	6.1
End Point	649	10.0	2.3	2.4	2.0	15.8	8.1	5.2	7.9	7.3
Corner	2260	116.0	3.0	10.1	2.0	125.5	8.6	12.8	8.3	25.6
L Line	186	7.0	2.0	1.6	1.5	13.0	7.7	4.0	7.4	5.7
T Junction	567	8.0	2.0	1.8	2.0	33.2	7.9	5.7	7.7	9.5
X Junction	35	6.0	1.7	1.0	1.0	10.8	7.9	2.5	7.5	4.7
Y Junction	188	7.0	2.0	1.5	2.0	16.6	7.6	4.1	7.4	8.8

Table B 8a - Statistics for Dark Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=15$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	608	22.0	5.0	15.7	4.0	48.1	23.1	40.5	22.7	7.8
Line	476	24.0	4.9	17.8	4.0	45.8	23.4	41.4	22.8	9.6
Wide Line	130	24.0	5.0	18.5	4.0	38.3	23.4	33.1	23.0	6.8
Right Angle	342	23.0	5.0	18.2	4.0	39.5	22.9	40.0	22.5	7.2
Acute Angle	341	30.0	4.9	15.4	4.0	62.2	23.3	38.2	22.5	7.0
End Point	775	28.0	6.1	23.0	5.0	48.6	24.2	44.2	23.6	8.4
Corner	2538	55.0	8.0	47.3	6.0	70.0	25.3	70.2	24.6	14.2
L Line	186	19.0	4.9	14.7	4.0	36.8	23.6	33.7	24.1	5.4
T Junction	708	24.0	5.1	16.2	4.0	45.6	23.4	42.0	23.0	6.1
X Junction	62	18.0	4.9	15.1	4.0	38.7	23.6	37.6	24.1	5.1
Y Junction	211	16.0	4.1	9.9	3.0	45.4	22.4	35.9	22.3	5.3

Table B 8b - Statistics for Bright Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=15$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max
Edge	570	22.0	5.0	17.8	4.0	37.0	22.9	31.3	22.7	8.6
Line	489	21.0	4.7	14.5	3.0	46.5	23.0	40.0	22.8	7.6
Wide Line	108	16.0	4.9	12.9	4.0	39.9	23.3	31.2	23.1	5.3
Right Angle	326	34.0	4.7	17.9	4.0	45.8	23.1	37.7	22.7	7.9
Acute Angle	326	20.0	5.2	16.1	4.0	42.8	23.4	42.5	22.9	7.4
End Point	795	35.0	6.1	26.2	5.0	51.4	24.6	51.3	24.0	8.9
Corner	2558	98.0	7.8	44.0	6.0	122.4	25.1	63.1	24.5	10.9
L Line	195	20.0	4.3	11.5	3.0	40.4	21.9	35.8	21.7	7.8
T Junction	750	26.0	5.3	18.7	4.0	43.3	23.2	42.2	22.9	6.8
X Junction	57	14.0	4.9	9.7	5.0	41.1	21.8	37.5	21.4	4.7
Y Junction	194	19.0	4.7	14.3	4.0	38.9	23.0	38.3	23.1	6.4

Table B 9a - Statistics for Dark Center Pixel Masks in a 64×64 "White Noise" Image with $\mu=127, \sigma=25$													
Class	# of Masks	Robustness			Contrast			Fisher distance					
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	571	34.0	8.0	43.2	6.0	73.7	37.6	92.1	37.2	9.8	2.8	0.8	2.7
Line	504	40.0	8.4	48.6	6.0	77.8	38.1	113.5	37.3	8.7	2.8	0.7	2.7
Wide Line	118	44.0	7.8	41.6	6.0	66.2	37.2	108.9	36.2	5.7	2.9	0.6	2.7
Right Angle	337	53.0	8.2	53.9	6.0	74.2	37.9	110.0	36.2	8.6	2.8	0.9	2.7
Acute Angle	349	37.0	8.3	48.9	6.0	70.7	38.4	124.7	37.7	7.0	2.8	0.7	2.6
End Point	821	46.0	9.5	61.0	7.0	86.2	39.4	133.6	38.4	11.9	2.6	0.7	2.4
Corner	2586	76.0	12.3	101.0	10.0	99.1	40.8	154.4	39.8	9.5	2.5	0.7	2.3
L Line	232	47.0	7.7	50.2	6.0	76.6	38.4	120.3	37.7	5.0	2.9	0.5	2.8
T Junction	781	41.0	8.0	47.7	6.0	76.6	38.0	116.4	36.9	9.0	2.7	0.6	2.6
X Junction	60	20.0	7.6	31.9	6.0	62.5	36.9	81.6	35.3	5.4	3.0	0.7	2.8
Y Junction	223	39.0	7.6	46.0	5.0	74.9	37.5	111.0	35.9	6.6	2.9	0.5	2.9

Table B. 9b - Statistics for Bright Center Pixel Masks in a 64×64 "White Noise" Image with $\mu=127$, $\sigma=25$													
Class	# of Masks	Robustness			Contrast			Fisher distance					
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	605	35.0	8.0	41.3	6.0	73.3	38.0	107.9	37.0	6.0	2.8	0.5	2.7
Line	506	34.0	7.7	38.0	6.0	82.8	37.9	107.6	37.3	6.4	2.7	0.5	2.6
Wide Line	101	37.0	8.7	53.0	6.0	76.1	38.7	97.5	37.4	6.5	3.0	0.9	2.8
Right Angle	358	38.0	7.6	41.8	6.0	66.3	37.2	96.5	36.7	7.3	2.8	0.6	2.6
Acute Angle	342	38.0	8.4	44.2	7.0	76.0	38.3	106.1	37.5	6.0	2.8	0.5	2.7
End Point	826	65.0	9.8	70.0	7.0	99.7	39.2	128.8	38.3	8.1	2.6	0.7	2.4
Corner	2651	95.0	12.3	118.6	9.0	123.4	41.6	174.6	40.3	7.8	2.5	0.7	2.3
L Line	223	32.0	7.1	33.7	5.0	66.2	37.4	103.3	36.8	5.9	2.8	0.5	2.7
T Junction	726	48.0	8.3	45.2	7.0	87.9	38.5	103.6	38.1	7.5	2.8	0.7	2.6
X Junction	58	36.0	8.3	52.5	7.0	62.3	36.2	91.8	36.2	8.7	3.2	1.4	2.9
Y Junction	220	32.0	7.5	40.3	5.0	69.4	38.6	105.3	37.4	7.0	2.9	0.6	2.8

Table B-10a - Statistics for Dark Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=30$

Class	# of Masks	Robustness				Contrast				Fisher distance			
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	627	50.0	8.4	58.1	6.0	83.2	44.4	148.5	43.2	9.1	2.8	0.6	2.6
Line	500	50.0	9.5	72.6	7.0	88.8	45.3	149.4	44.0	7.2	2.8	0.7	2.6
Wide-Line	92	36.0	9.2	54.0	7.0	76.9	44.4	128.8	44.2	5.5	2.9	0.6	2.7
Right-Angle	382	56.0	9.2	55.9	7.0	81.2	44.8	143.8	43.2	7.8	2.8	0.6	2.7
Acute-Angle	358	53.0	10.2	67.1	8.0	92.5	46.0	152.5	46.3	8.9	2.9	0.8	2.7
End-Point	810	55.0	11.3	93.9	9.0	103.7	47.3	177.3	46.2	9.1	2.6	0.7	2.4
Corner	2637	101.0	15.1	182.1	11.0	120.6	49.2	251.0	47.4	14.0	2.5	0.9	2.3
L-Line	223	45.0	9.0	66.3	6.0	86.3	46.0	125.5	45.8	7.9	2.9	0.8	2.7
T-Junction	779	54.0	10.0	70.9	8.0	88.1	45.7	148.9	44.2	8.5	2.7	0.7	2.6
X-Junction	69	46.0	8.3	79.4	6.0	76.6	42.7	153.2	41.1	5.9	2.7	0.7	2.6
Y-Junction	232	51.0	9.5	77.9	7.0	82.0	43.4	146.8	42.4	7.5	3.0	0.8	2.7

Table B-10b - Statistics for Bright Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=30$

Class	# of Masks	Robustness				Contrast				Fisher distance			
		Max	Mean	Var	Med	Max	Mean	Var	Med	Max	Mean	Var	Med
Edge	579	50.0	8.8	64.1	6.0	81.8	44.1	152.8	44.0	7.4	2.8	0.7	2.7
Line	528	45.0	9.2	64.4	6.5	95.3	44.7	157.3	43.5	6.6	2.8	0.6	2.6
Wide-Line	107	48.0	8.8	59.3	7.0	83.9	44.2	167.6	43.1	7.3	2.9	0.8	2.8
Right-Angle	361	46.0	9.2	61.2	7.0	84.2	45.1	162.6	43.8	6.5	2.7	0.6	2.6
Acute-Angle	326	49.0	9.5	77.0	7.0	98.5	44.6	180.6	43.3	7.0	2.8	0.8	2.6
End-Point	824	69.0	11.1	105.8	8.0	100.3	46.4	195.2	45.1	11.6	2.6	0.9	2.4
Corner	2596	86.0	14.8	160.2	11.0	123.0	48.9	239.0	47.3	19.8	2.5	0.9	2.3
L-Line	238	42.0	8.5	56.7	6.0	79.8	44.5	159.4	44.6	5.9	2.9	0.5	2.8
T-Junction	800	55.0	10.0	71.6	8.0	88.9	45.0	162.4	44.3	9.2	2.7	0.7	2.6
X-Junction	66	27.0	8.2	42.0	6.0	68.9	42.4	102.1	43.2	6.3	2.9	0.7	2.9
Y-Junction	247	48.0	8.5	56.9	6.0	86.4	43.1	144.0	42.3	6.7	2.9	0.6	2.7

Table B-11a - Statistics for Dark Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=35$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Med
Edge	647	56.0	10.1	72.5	101.7	51.1	198.9	7.5	2.8	2.7
Line	535	49.0	10.8	82.1	97.8	52.8	203.4	7.1	2.8	2.7
Wide-Line	120	43.0	9.6	57.0	88.8	51.7	166.6	5.6	2.9	2.8
Right-Angle	334	52.0	9.9	74.1	92.7	51.0	198.4	7.6	2.8	2.6
Acute-Angle	352	58.0	10.6	100.3	103.5	53.2	233.1	6.6	2.8	2.7
End-Point	823	63.0	13.5	131.2	104.3	54.6	256.4	9.2	2.7	2.5
Corner	2657	89.0	16.7	214.2	129.9	56.0	318.7	8.2	2.5	2.3
L-Line	233	43.0	10.8	75.8	99.3	51.6	208.4	7.0	2.8	2.7
T-Junction	757	67.0	11.9	110.8	119.3	53.0	238.2	7.1	2.7	2.6
X-Junction	49	27.0	7.3	26.4	84.6	50.8	176.0	3.8	2.6	2.6
Y-Junction	224	47.0	11.4	98.3	100.1	51.2	226.5	7.8	2.9	2.8

Table B-11b - Statistics for Bright Center Pixel Masks in a 64X64 "White Noise" Image with $\mu=127$, $\sigma=35$

Class	# of Masks	Robustness			Contrast			Fisher distance		
		Max	Mean	Var	Max	Mean	Var	Max	Mean	Med
Edge	601	58.0	11.0	89.3	98.7	53.7	198.5	7.0	2.8	2.7
Line	542	61.0	10.1	87.1	95.3	52.7	189.6	9.9	2.7	2.6
Wide-Line	127	39.0	9.1	55.1	95.4	50.6	217.7	6.5	2.8	2.7
Right-Angle	339	44.0	10.0	63.1	104.8	51.7	210.5	7.1	2.7	2.5
Acute-Angle	351	48.0	10.5	78.9	98.3	50.4	185.6	7.4	2.7	2.6
End-Point	841	66.0	12.8	121.5	115.0	53.7	254.8	11.2	2.6	2.4
Corner	2645	97.0	18.0	256.9	147.3	57.7	346.3	8.8	2.6	2.4
L-Line	235	41.0	9.9	68.5	86.1	50.5	175.8	6.6	2.8	2.6
T-Junction	730	63.0	11.6	101.6	114.3	52.6	227.1	7.7	2.7	2.6
X-Junction	62	24.0	6.8	28.7	78.7	46.5	128.9	5.0	2.8	2.7
Y-Junction	225	44.0	10.1	71.6	90.7	52.8	166.6	6.3	2.8	2.7

END

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